Impact on Financial Markets of Dark Pools, Large Investor, and HFT

Shin Nishioka¹, Kiyoshi Izumi¹, Wataru Matsumoto², Takashi Shimada¹, Hiroki Sakaji¹, and Hiroyasu Matushima¹

¹ Graduate School of Engineering, The University of Tokyo, 7-3-1 Hongo, Bunkyo, Tokyo, Japan
² Nomura Securities Co., Ltd.*
izumi@sys.t.u-tokyo.ac.jp

Abstract. In this research, we expanded an artificial market model including the lit market, the dark pools, the large investor, and the market maker (HFT). Using the model, we investigated their influences on the market efficiency and liquidity. We found that dark pools may improve the market efficiency if their usage rate were under some threshold. Especially, it is desirable that the main users of the dark pool are large investors. A certain kind of HFT such as the market making strategy may provide the market liquidity if their usage rate were under some threshold.

Keywords: Artificial Market - Dark pool - Large investor.

1 Introduction

Dark pools, private financial forums for trading securities, are becoming widely used in finance especially by institutional investors [15]. Dark pools allow investors to trade without showing their orders to anyone else.

One of the main advantages of dark pools is their function to significantly reduce the market impact of large orders. Large investors have to constantly struggle with the problem that the market price moves adversely when they buy or sell large blocks of securities. Such market impacts are considered as trading costs for large investors. Hiding the information of large orders in dark pools may decrease market impacts. Moreover, from the viewpoint of a whole market, dark pools may stabilize financial markets by reducing market impacts[7].

On the other hand, one of the main disadvantages of dark pools is the lack of transparency of their market information. The lack of transparency could result in the damage of price discovery function of the lit markets, a public exchange that provide all order books to investors. That may cause market instability and inefficiency[4, 17]. Therefore, for example in Europe, regulators are discussing introducing a volume cap regulation for dark pools, which means that they should be subject to limits on what volume of orders can be traded in them[16].

* It should be noted that the opinions contained herein are solely those of the authors and do not necessarily reflect those of Nomura Securities Co., Ltd.
High frequency trading (HFT) is another trend in finance. HFT is a type of
electronic program trading that uses computers to transact a large number of
orders at high speeds. In 2014, HFT in the United States is expected to account
for approximately 48.5\% of exchange trading volume \cite{1}. Dark pools account for
13.7\% of the total trading volume\cite{14}. HFT firms participate in both lit markets
and dark pools.

The effect of interaction between HFT and dark pools on market stability
and efficiency is becoming increasingly important issue. Some researchers are
concerned the issue that the heavily use of dark pools by HFT firms causes the
divergence of dark pools from lit markets. Then, the price discovery process can
begin to erode\cite{10}. It is however very difficult to discuss what are caused by
the interaction of HFT and dark pools by using only results of empirical studies
because some situations have never occurred before in real financial markets.

An artificial market, which is a kind of a multi-agent simulation, will help us
to discuss situations that have never occurred before in real financial markets\cite{9,
2}. Using artificial market simulations, many studies have investigated effects
of market segmentation and changing regulations\cite{18,12}. A previous study\cite{13}
investigated dark pools using artificial market simulations. However, because
they also use historical real financial stock prices, they have not investigated
situations that have never occurred before, such as usage rates of dark pools
that are much higher than those at present.

We built a simple artificial market model on the basis of the model of Mizuta
et al. \cite{11} including one lit market and one dark pool. Using the model, we intro-
duced algorithm agents and investigated whether dark pools stabilize markets
or not by observing market impacts of these agents. We emphasize that these
investigations included cases in which usage rates of dark pools are much higher
than those of present financial markets.

\section{Artificial market model}

The artificial market model of this study is based on Mizuta et al. \cite{11} and
Chiarella et al. \cite{3}. Chiarella et al. modeled only a lit market and Mizuta et al.
added a dark pool to \cite{3}. Our model added HFT firms and large investors to
Mizuta et al. \cite{11}. Those preceding models succeeded at replicating stylized facts
that are the statistical nature of long-term returns that stably exist in almost
all financial markets. The model of Mizuta et al. succeed at replicating high
frequency micro structures such as the trade rate, cancel rate, one tick volatility,
and so on, which were not replicated by the model of Chiarella et al.

\subsection{Markets}

Figure 1 shows the overall view of the model of this study. The model treats only
one risk asset and non-risk asset (cash). In the model, there are three markets:
One lit market, which provides all order books to investors, and two dark pools,
which provide no order books. One dark pool accepts orders from HFT firms, but the other dark pool does not accept.

This model consists of three types of agents: 100 stylized traders (normal traders), one large investor, and one HFT agent. In each time step $t$, one stylized trader $j$ is randomly selected (Fig. 2). This stylized trader places an order to buy or sell the risk asset: For each time, each agent $j$ determines its bid or ask limit price $P_{o;j}(t)$. Stylized traders always order only one share and can short-sell freely. The quantity of holding positions is not limited, so agents can take any shares for both long and short positions to infinity. After the stylized trader’s order, the larger investor and HFT agent place their orders with their respective probabilities. This cycle is repeated. Note that time $t$ passes even if no deals are done.

**Lit market** The lit market adopts a continuous double auction to determine a market price of the risk asset. A continuous double auction is an auction mechanism where multiple buyers and sellers compete to buy and sell some financial assets in the market, and where transactions can occur at any time whenever an offer to buy and an offer to sell match [5, 6]. In the lit market, transactions are made by matching against relative orders according to price and time priority principle (Fig. 3).

All agents can acquire the information of the lit market: Transaction prices of the lit market $P_{lit}(t)$; best bid prices $P_{bb}(t)$; best ask prices $P_{ba}(t)$; mid prices $P_{mid}(t) = (P_{bb}(t) + P_{ba}(t))/2$. The minimum unit, a tick size, of the lit market price $P_{lit}(t)$’s change is $\delta P = 0.1$. The buy order price is rounded off to the nearest fraction, and the sell order price is rounded up to the nearest fraction. When an agent orders to buy (sell), if there is a lower price sell order (a higher price buy order) than the agent’s order, dealing is immediately done, we call this a market order. If there is not, the agent’s order remains in the order book, we call this a limit order. The remaining order is canceled $t_c$ after the order time.
Contrary to the lit market, agents cannot get the information such as transaction prices and order prices in the dark pools. There are many ways to determine trade prices in dark pools. In the model, the dark pool adopts a mid price of the lit market \( P_{\text{mid}}(t) \), an average price of the best bid and offer in the lit market, as its trade price (Fig. 3). This method is adopted by many dark pools in real financial markets [2]. Agents do not specify an order price in the dark pool. When an agent orders one unit buy (sell) order, trading is done immediately if there are opposite sell (buy) orders. If there are no opposite orders, the order remains and waits for opposite orders to come. In the dark pool, therefore, only either buy or sell orders remain. The same as in a lit market, the remaining order is canceled \( t_c \) after the order time.

2.2 Agents

An agent of an artificial market is a virtual trader that places its buying or selling orders to financial markets in the model. There are 3 types of agents in our model: 100 stylized traders (normal traders), one large investor, and one HFT agent.

Stylized trader Stylized traders (normal agents) in the model are the same as agents in the model of Mizuta et al.[11]. Stylized traders need to replicate characteristics of price formations in real financial markets, stylized facts, and market microstructures.

A stylized trader \( j \) determines an order price \( P_{o,j}(t) \) and buys or sells by the following process. It uses a combination of fundamental value and technical rules to form expectations on a risk asset returns \( r_{e,j}(t) \)
where $w_{i,j}$ is a weight of term $i$ of the agent $j$ and is determined by random variables uniformly distributed in the interval $(0, w_{i,\text{max}})$ at the start of the simulation independently for each agent. $P_f(t)$ is a fundamental value that is constant. $P_{\text{lit}}(t)$ is a lit market price of the risk asset at time $t$. When the dealing is not done at $t$, $P_{\text{lit}}(t)$ remains at the last market price $P_{\text{lit}}(t-1)$, and at $t = 1$, $P_{\text{lit}}(t) = P_f(t)$. $\epsilon_j(t)$ is a noise determined by random variables of normal distribution with an average 0 and a variance $\sigma$. $r_{h,j}$ is a historical price return inside an agent’s time interval $\tau_j$, and $r_{h,j} = \log \left( \frac{P_{\text{lit}}(t)}{P_{\text{lit}}(t-\tau_j)} \right)$. $\tau_j$ is determined by random variables uniformly distributed in the interval $[\tau_{\text{min}}, \tau_{\text{max}}]$ at the start of the simulation independently for each agent. The first term of Eq. 1 represents a fundamental strategy: an agent expects a positive return when the market price is lower than the fundamental value, and vice versa. The second term of Eq. 1 represents a technical strategy: an agent expects a positive return when the historical market return is positive, and vice versa. After the expected log return $r_{e,j}(t)$ has been determined by the equation 1, an expected price $P_{o,j}(t)$ is determined as follows.

$$r_{e,j}(t) = \frac{1}{w_{1,j} + w_{2,j} + w_{3,j}} \left( w_{1,j} \log \frac{P_f(t)}{P_{\text{lit}}(t)} + w_{2,j} r_{h,j}(t) + w_{3,j} \epsilon_j(t) \right),$$  \hspace{1cm} (1)$$

$$P_{o,j}(t) = P_{\text{lit}}(t) \cdot \exp (r_{e,j}(t))$$  \hspace{1cm} (2)$$

We modeled an order price by random variables of normal distributed in an average $P_{o,j}(t)$, a standard deviation $P_\sigma$, where $P_\sigma$ is a constant. A minimum unit of a price change (tick size) is $\delta P$, we round off a fraction of less than $\delta P$. 

**Fig. 3. Lit Market and Dark Pool**
Buy or sell is determined by a magnitude relationship between the expect price \( P_e;j(t) \) and the order price \( P_o;j(t) \), that is,

- When \( P_e;j(t) > P_o;j(t) \), the agent orders to buy one share.
- When \( P_e;j(t) < P_o;j(t) \), the agent orders to sell one share.

The stylized traders order in the lit market with a probability \( 1 - d_s \), in the dark pool with HFT with a probability \( 0.5d_s \), and in the dark pool without HFT with a probability \( 0.5d_s \). An order of a stylized trader is canceled if the order is not executed within the \( t^*_c \) steps.

**Large investor** A large investor determine similarly its order price and direction to buy or sell as a stylized trader. However, it differs from the stylized trader in the way of ordering: Slicing of large orders. After the expectation, the large investor start to place orders in \( \mu \) units with a possibility \( p \) and continues to place \( \mu \) units' orders in \( k \) time steps. The order prices, the direction (buy or sell), and the selected market do not change in those \( k \) steps. The order volume of the large investor is \( \mu \times k \) times larger than that of the stylized trader. In this study, the parameters are set as \( (p, \mu, k) = (0.05, 5, 5) \). The large investor orders in the lit market with a probability \( 1 - d_L \), in the dark pool with HFT with a probability \( 0.5d_L \), and in the dark pool without HFT with a probability \( 0.5d_L \).

**HFT agent (market maker, MM)** In this model, we introduced a market maker (MM)\[8\], an agent with a market making strategy that is one of major trading strategies of HFT. HFT, although not strictly defined at present, has roughly the following characteristics. It is a trading strategy that aims for small profits by placing orders automatically by a computer program to utilize the price difference in a very short time such as milliseconds and microseconds. There are some kinds of the trading strategy of HFT, and a market-making strategy occupies a large percentage of them. A market-making strategy places both buying and selling orders near a middle price by high frequency. Although it can provide the liquidity of trading, it may distort price formation.

MM can place its order just after each stylized trader placed its order in every time steps. That is, MM has much higher speed of ordering than the stylized trader. The trading strategy of MM consists of the following steps. Firstly if there is MM’s own order in the markets, MM cancels them. Secondly, MM places one unit of buying order at a lower price \( P_{buy,MM}(t) \) and one unit of selling order at a higher price \( P_{sell,MM}(t) \) simultaneously (Fig. 4).

\[
P_{buy,MM}(t) = \begin{cases} 
P_{ba}(t) & (P_{fair,MM}(t) \geq P_{bb}(t) + 0.5P_f \theta) \\
P_{fair,MM}(t) - 0.5P_f \theta (\text{others}) & (P_{fair,MM}(t) \leq P_{ba}(t) - 0.5P_f \theta) \\
P_{bb}(t) - P_f \theta & \end{cases} \tag{3}
\]

\[
P_{sell,MM}(t) = \begin{cases} 
P_{ba}(t) + P_f \theta & (P_{fair,MM}(t) \geq P_{bb}(t) + 0.5P_f \theta) \\
P_{fair,MM}(t) + 0.5P_f \theta (\text{others}) & (P_{fair,MM}(t) \leq P_{ba}(t) - 0.5P_f \theta) \\
P_{bb}(t) & \end{cases} \tag{4}
\]
\[ P_{\text{fair,MM}}(t) = (1 - w_{MM} \cdot s(t)^3)P_{\text{mid}}(t) \] (5)

The equations 3-4 means the position control of MM. That is, MM is willing to sell (buy) at a lower selling price \( P_{\text{sell,MM}}(t) \) (higher buying price \( P_{\text{buy,MM}}(t) \)) when it has larger positive (negative) position \( s(t) \) respectively. In this study, the value of \( w_{MM} \) is set to \( 2.0 \times 10^{-6} \).

MM usually uses the lit market for its orders. It however uses the HFT-accepted dark pool when the following two conditions are satisfied:

1. The selling price \( P_{\text{sell,MM}}(t) \) (buying price \( P_{\text{buy,MM}}(t) \)) reaches the minimum price \( P_{bb}(t) \) (the maximum price \( P_{ba}(t) \)) because of its large positive (negative) position respectively.
2. There are orders on the opposite side and MM’s order can be transacted in the dark pool.

3 Simulation results

Using our model, we examined the influence on the market of the interaction among the dark pools, the large investor, and HFT.

3.1 Market efficiency and liquidity

We used the market inefficiency and liquidity as the indicators to measure the influence on the market. The market inefficiency \( M_{ie} \) is calculated based on the
difference between the fundamental price $P_f(t)$ and the lit market price $P_{lit}(t)$ [11]:

$$M_{ic} = \frac{1}{T} \sum_{t=t_s}^{t_s+T-1} \frac{|P_{lit}(t) - P_f(t)|}{P_f(t)},$$

(6)

where $t_s$ is the time step when the analysis period starts after the preparation period of simulation. $T$ is the duration of the analysis period. In this study, $t_s = 500, T = 10,000$. If the divergence of $P_{lit}(t)$ from $P_f(t)$ is larger, the market is more inefficient.

The liquidity $L_m$ of market $m$={lit, dark pool w/HFT, dark pool wo/HFT} is defined based on the transaction rate:

$$L_m = \frac{n_{trans,m}}{n_{order,m}},$$

(7)

where $n_{trans,m}$ is a number of all transactions and $n_{order,m}$ is a number of all orders in market $m$ during $t = (t_s, t_s + T - 1)$.

### 3.2 Parameter setting

In this study, values of the parameters are set as tables 1-4. These values were determined with reference to [11]. Under these parameter values, our model could replicate both long-term statistical characteristics (the fat-tail of return distribution and volatility clustering) and short-term market micro structures (transaction rates and cancel rates) of real financial markets.

**Table 1. Parameters of markets**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start of analysis period</td>
<td>$t_s = 500$</td>
</tr>
<tr>
<td>Duration of analysis periods</td>
<td>$T = 10,000$</td>
</tr>
<tr>
<td>Tick size</td>
<td>$\delta P = 1$</td>
</tr>
<tr>
<td>Fundamental price</td>
<td>$P_f = 10,000$</td>
</tr>
</tbody>
</table>

**Table 2. Parameters of stylized trader**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of stylized traders</td>
<td>$n = 100$</td>
</tr>
<tr>
<td>Maximum weight of fundamental term</td>
<td>$w_{1,max} = 1.0$</td>
</tr>
<tr>
<td>Maximum weight of chartist term</td>
<td>$w_{2,max} = 0.1$</td>
</tr>
<tr>
<td>Maximum weight of noise term</td>
<td>$w_{3,max} = 1.0$</td>
</tr>
<tr>
<td>Minimum time interval of chart analysis</td>
<td>$\tau_{min} = 100$</td>
</tr>
<tr>
<td>Maximum time interval of chart analysis</td>
<td>$\tau_{max} = 200$</td>
</tr>
<tr>
<td>Standard deviation of noise term</td>
<td>$\sigma_c = 0.001$</td>
</tr>
<tr>
<td>Standard deviation of expected price</td>
<td>$P_e = [0, 0.1]$</td>
</tr>
</tbody>
</table>
Impact on Financial Markets of Dark Pools, Large Investor, and HFT

Table 3. Parameters of large investor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of large investor</td>
<td>$n_{large} = 1$</td>
</tr>
<tr>
<td>Maximum weight of fundamental term</td>
<td>$w_{1,max} = 1.0$</td>
</tr>
<tr>
<td>Maximum weight of chartist term</td>
<td>$w_{2,max} = 0.1$</td>
</tr>
<tr>
<td>Maximum weight of noise term</td>
<td>$w_{3,max} = 1.0$</td>
</tr>
<tr>
<td>Time interval of chart analysis</td>
<td>$\tau_j = 150$</td>
</tr>
<tr>
<td>Standard deviation of noise term</td>
<td>$\sigma = 0.001$</td>
</tr>
<tr>
<td>Standard deviation of expected price</td>
<td>$P_e = [0, 0.1]$</td>
</tr>
<tr>
<td>Probability of slicing start</td>
<td>$p = 0.05$</td>
</tr>
<tr>
<td>Order size of each order</td>
<td>$\mu = 5$</td>
</tr>
<tr>
<td>Number of slicing</td>
<td>$k = 5$</td>
</tr>
</tbody>
</table>

Table 4. Parameters of market maker

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of market maker</td>
<td>$n_{MM} = 1$</td>
</tr>
<tr>
<td>Spread rate</td>
<td>$\theta = 0.001$</td>
</tr>
<tr>
<td>Coefficient of position control</td>
<td>$w_{MM} = 0.000002$</td>
</tr>
<tr>
<td>Initial position</td>
<td>$s(0) = 0$</td>
</tr>
<tr>
<td>Duration of holding orders</td>
<td>$t_C = 1$</td>
</tr>
</tbody>
</table>

3.3 Market efficiency results

First, we examined the influence on the market efficiency of the dark pool without HFT. Figure 5 shows the relationship between the market inefficiency $M_{ie}$, the probability of dark pool orders by the stylized traders $d_S$, and the probability of dark pool orders by the large investor $d_L$. The more the stylized traders use the dark pool (the larger $d_S$), the less efficient the market is (the larger $M_{ie}$). On the contrary, the more the large investor uses the dark pool (the larger $d_L$), the more efficient the market is (the smaller $M_{ie}$), as Figure 6 shows. Next, we also examined the change of market efficiency with HFT. The market efficiency with HFT is larger than that without HFT.

We considered the mechanism of those two results is as follows:

1. When too many stylized traders use the dark pool ($d_s$ is too large), the number and volume of orders in the lit market become too small. Then the price changes caused by one order become large and the lit market becomes unstable. That is, the market price $P_{lit}(t)$ tends to derive from the fundamental price $P_f(t)$ and the market inefficiency $M_{ie}$ becomes larger.

2. When the large investor uses the lit market, its large order at a certain price disturbs the free price movement and sticks the market price $P_{lit}(t)$ away from the fundamental price $P_f(t)$. Thus, the large investor uses the dark pool ($d_L$ is larger), the market price can return to the fundamental price level more easily and the market becomes more efficient ($M_{ie}$ becomes smaller).

3. When HFT exists in a lit market, there are both selling and buying orders around the mid price constantly. Those orders block the market price’s deviation from the fundamental price.
Fig. 5. Market inefficiency and the probability of dark pool orders by the stylized traders $d_S$

3.4 Market liquidity results

In this study, HFT trader is the market maker (MM). MM provides many orders to the markets and makes it easier to match orders. Thus, the liquidity (transaction rate) of the dark pool with HFT $L_{\text{dark pool w/HFT}}$ tends to be larger than the liquidity of the dark pool without HFT $L_{\text{dark pool wo/HFT}}$. We examined the change of the difference between those liquidity $\Delta L$ when the percentage of using the dark pools by the stylized traders $d_S$ changed.

$$\Delta L = L_{\text{dark pool w/HFT}} - L_{\text{dark pool wo/HFT}}$$

(8)

Figure 7 shows the relationship between $\Delta L$ and $d_S$. The more stylized traders use the dark pools, the smaller the difference becomes. That is, the stylized trader’s use of the dark pools reduced MM’s contribution to the market liquidity.

We also examined the percentages of MM’s use of the dark pool in changing the percentages of the stylized traders’ use of the dark pools $d_S$. The results shows that the more the stylized traders use the dark pools, the less MM uses the dark pool. The decrease of MM’s use of the dark pool reduces the liquidity advantage of the HFT-accepted dark pool in comparison with the HFT-prohibited dark pool.

The mechanism of those results is considered as follows. MM usually uses the lit market. It however uses the HFT accepted dark pool when it has a large positive or negative amount of assets and there are orders on the opposite
Fig. 6. Market inefficiency and the probability of dark pool orders by the large investor $d_L$.

side in the dark pool. The simulation data shows that the rate at which the first condition (the large amount of assets) was satisfied was almost constant at every utilization rate of the stylized traders $d_S$. On the contrary, the rate at which the second condition (the existence of opposite orders in the dark pool) is satisfied decreases as $d_S$ increases. When the stylized traders uses the dark pools frequently ($d_S$ is larger), the orderbooks of the dark pools become similar to the orderbook of the lit market. Thus, the unbalance of orders and the absence of opposite orders tend to appear both in the lit market and the dark pools. That is, the second condition is not satisfied more frequently.

4 Conclusions

In this research, we expanded an artificial market model on the bases of the model of [11, 8] including the lit market, the dark pools, the large investor, and the market maker (HFT). Using the model, we investigated their influences on the market efficiency and liquidity.

We found that too much usage of the dark pool by the stylized traders (normal traders) would lead to the inefficiency of the market (the large deviation from the fundamental price). This result is similar to the preceding study’s results[11]. On the contrary, the large investor’s usage of the dark pool made the market more efficient. The market maker (HFT) improved the market liquidity (the transaction rate of orders). However, the increasing usage of the dark pool
Fig. 7. Difference of liquidity and the probability of dark pool orders by the stylized traders $d_S$

by the stylized traders reduced the improvement rate of the market liquidity by HFT.

These results suggested that dark pools may improve the market efficiency if their usage rate were under some threshold. Especially, it is desirable that the main users of the dark pool are large investors. A certain kind of HFT such as the market making strategy may provide the market liquidity if their usage rate were under some threshold.

Acknowledgments

This research was supported by KAKENHI (no. 15H02745) and MEXT via Exploratory Challenges on Post-K computer (Study on multilayered multiscale space-time simulations for social and economic phenomena).

References