

Impact on Financial Markets of Dark Pools, Large Investor, and HFT

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Abstract. In this research, we expanded an artificial market model including the lit market, the dark pools, the large investor, and the market maker (HFT). Using the model, we investigated their influences on the market efficiency and liquidity. We found that dark pools may improve the market efficiency if their usage rate were under some threshold. Especially, It is desirable that the main users of the dark pool are large investors. A certain kind of HFT such as the market making strategy may provide the market liquidity if their usage rate were under some threshold.

Keywords: Artificial Market · Dark pool · Large investor.

1 Introduction

Dark pools, private financial forums for trading securities, are becoming widely used in finance especially by institutional investors [15]. Dark pools allow investors to trade without showing their orders to anyone else.

One of the main advantages of dark pools is their function to significantly reduce the market impact of large orders. Large investors have to constantly struggle with the problem that the market price moves adversely when they buy or sell large blocks of securities. Such market impacts are considered as trading costs for large investors. Hiding the information of large orders in dark pools may decrease market impacts. Moreover, from the viewpoint of a whole market, dark pools may stabilize financial markets by reducing market impacts[7].

On the other hand, one of the main disadvantages of dark pools is the lack of transparency of their market information. The lack of transparency could result in the damage of price discovery function of the lit markets, a public exchange that provide all order books to investors. That may cause market instability and inefficiency[4, 17]. Therefore, for example in Europe, regulators are discussing introducing a volume cap regulation for dark pools, which means that they should be subject to limits on what volume of orders can be traded in them[16].

* It should be noted that the opinions contained herein are solely those of the authors and do not necessarily reflect those of Nomura Securities Co., Ltd.

High frequency trading (HFT) is another trend in finance. HFT is a type of electronic program trading that uses computers to transact a large number of orders at high speeds. In 2014, HFT in the United States is expected to account for approximately 48.5% of exchange trading volume [1]. Dark pools account for 13.7% of the total trading volume[14]. HFT firms participate in both lit markets and dark pools.

The effect of interaction between HFT and dark pools on market stability and efficiency is becoming increasingly important issue. Some researchers are concerned the issue that the heavily use of dark pools by HFT firms causes the divergence of dark pools from lit markets. Then, the price discovery process can begin to erode[10]. It is however very difficult to discuss what are caused by the interaction of HFT and dark pools by using only results of empirical studies because some situations have never occurred before in real financial markets.

An artificial market, which is a kind of a multi-agent simulation, will help us to discuss situations that have never occurred before in real financial markets[9, 2]. Using artificial market simulations, many studies have investigated effects of market segmentation and changing regulations[18, 12]. A previous study[13] investigated dark pools using artificial market simulations. However, because they also use historical real financial stock prices, they have not investigated situations that have never occurred before, such as usage rates of dark pools that are much higher than those at present.

We built a simple artificial market model on the basis of the model of Mizuta et al. [11] including one lit market and one dark pool. Using the model, we introduced algorithm agents and investigated whether dark pools stabilize markets or not by observing market impacts of these agents. We emphasize that these investigations included cases in which usage rates of dark pools are much higher than those of present financial markets.

2 Artificial market model

The artificial market model of this study is based on Mizuta et al. [11] and Chiarella et al. [3]. Chiarella et al. modeled only a lit market and Mizuta et al. added a dark pool to [3]. Our model added HFT firms and large investors to Mizuta et al. [11]. Those preceding models succeeded at replicating stylized facts that are the statistical nature of long-term returns that stably exist in almost all financial markets. The model of Mizuta et al. succeed at replicating high frequency micro structures such as the trade rate, cancel rate, one tick volatility, and so on, which were not replicated by the model of Chiarella et al.

2.1 Markets

Figure 1 shows the overall view of the model of this study. The model treats only one risk asset and non-risk asset (cash). In the model, there are three markets: One lit market, which provides all order books to investors, and two dark pools,

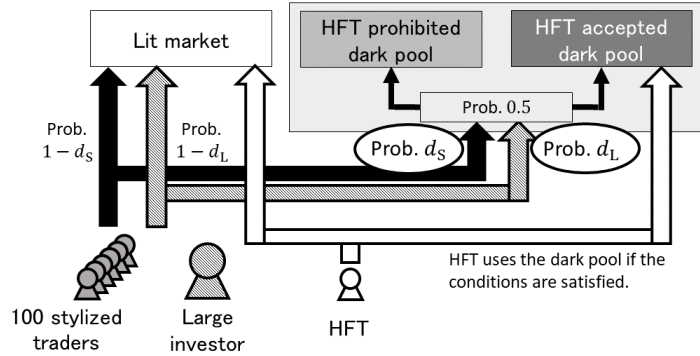


Fig. 1. Framework of Artificial Market

which provide no order books. One dark pool accepts orders from HFT firms, but the other dark pool does not accept.

This model consists of three types of agents: 100 stylized traders (normal traders), one large investor, and one HFT agent. In each time step t , one stylized trader j is randomly selected (Fig. 2). This stylized trader places an order to buy or sell the risk asset: For each time, each agent j determines its bid or ask limit price $P_{o,j}(t)$. Stylized traders always order only one share and can short-sell freely. The quantity of holding positions is not limited, so agents can take any shares for both long and short positions to infinity. After the stylized trader's order, the larger investor and HFT agent place their orders with their respective probabilities. This cycle is repeated. Note that time t passes even if no deals are done.

Lit market The lit market adopts a continuous double auction to determine a market price of the risk asset. A continuous double auction is an auction mechanism where multiple buyers and sellers compete to buy and sell some financial assets in the market, and where transactions can occur at any time whenever an offer to buy and an offer to sell match [5, 6]. In the lit market, transactions are made by matching against relative orders according to price and time priority principle (Fig. 3).

All agents can acquire the information of the lit market: Transaction prices of the lit market $P_{lit}(t)$; best bid prices $P_{bb}(t)$; best ask prices $P_{ba}(t)$; mid prices $P_{mid}(t) = (P_{bb}(t) + P_{ba}(t))/2$. The minimum unit, a tick size, of the lit market price $P_{lit}(t)$'s change is $\delta P = 0.1$. The buy order price is rounded off to the nearest fraction, and the sell order price is rounded up to the nearest fraction. When an agent orders to buy (sell), if there is a lower price sell order (a higher price buy order) than the agent's order, dealing is immediately done, we call this a *market order*. If there is not, the agent's order remains in the order book, we call this a *limit order*. The remaining order is canceled t_c after the order time.

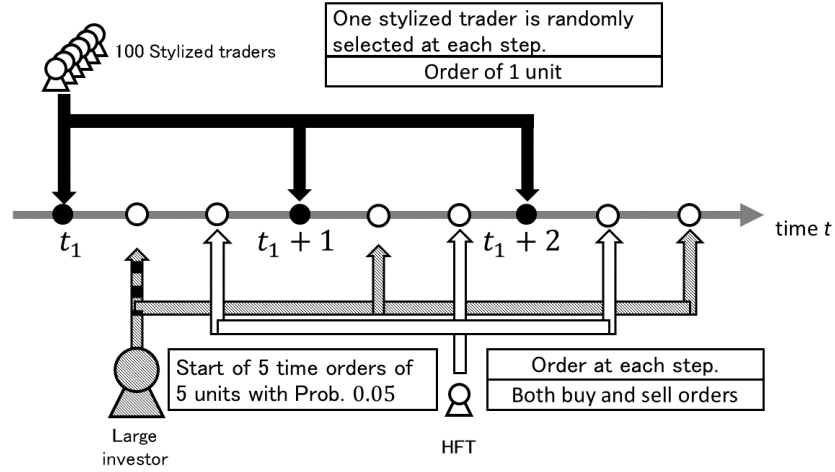


Fig. 2. Order processing

Dark pools Contrary to the lit market, agents can not get the information such as transaction prices and order prices in the dark pools. There are many ways to determine trade prices in dark pools. In the model, the dark pool adopts a mid price of the lit market $P_{mid}(t)$, an average price of the best bid and offer in the lit market, as its trade price (Fig. 3). This method is adopted by many dark pools in real financial markets [2]. Agents do not specify an order price in the dark pool. When the agent orders one unit buy (sell) order, trading is done immediately if there are opposite sell (buy) orders. If there are no opposite orders, the order remains and waits for opposite orders to come. In the dark pool, therefore, only either buy or sell orders remain. The same as in a lit market, the remaining order is canceled t_c after the order time.

2.2 Agents

An agent of an artificial market is a virtual trader that places its buying or selling orders to financial markets in the model. There are 3 types of agents in our model: 100 stylized traders (normal traders), one large investor, and one HFT agent.

Stylized trader Stylized traders (normal agents) in the model are the same as agents in the model of Mizuta et al.[11]. Stylized traders need to replicate characteristics of price formations in real financial markets, stylized facts, and market micro structures.

A stylized trader j determines an order price $P_{o,j}(t)$ and buys or sells by the following process. It uses a combination of fundamental value and technical rules to form expectations on a risk asset returns $r_{e,j}(t)$.

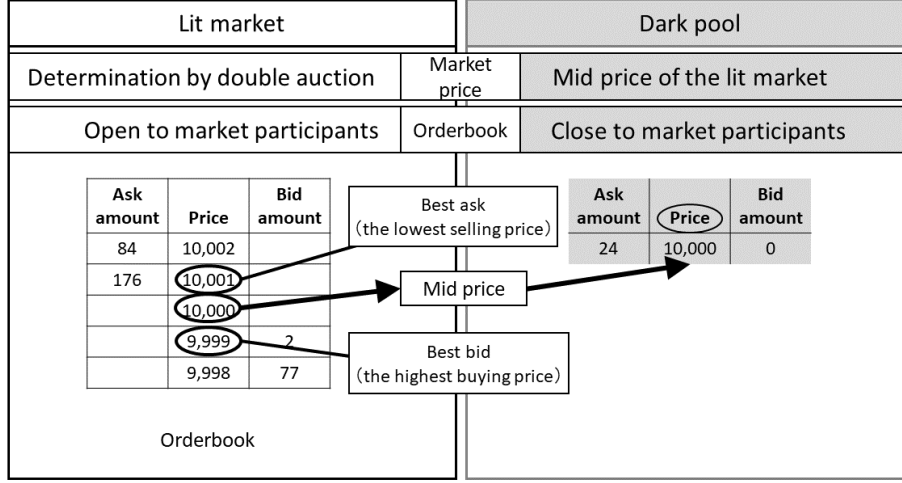


Fig. 3. Lit Market and Dark Pool

$$r_{e,j}(t) = \frac{1}{w_{1,j} + w_{2,j} + w_{3,j}} \left(w_{1,j} \log \frac{P_f(t)}{P_{lit}(t)} + w_{2,j} r_{h,j}(t) + w_{3,j} \epsilon_j(t) \right), \quad (1)$$

where $w_{i,j}$ is a weight of term i of the agent j and is determined by random variables uniformly distributed in the interval $(0, w_{i,max})$ at the start of the simulation independently for each agent. $P_f(t)$ is a fundamental value that is constant. $P_{lit}(t)$ is a lit market price of the risk asset at time t . When the dealing is not done at t , $P_{lit}(t)$ remains at the last market price $P_{lit}(t-1)$, and at $t=1$, $P_{lit}(t) = P_f(t)$. $\epsilon_j(t)$ is a noise determined by random variables of normal distribution with an average 0 and a variance σ . $r_{h,j}$ is a historical price return inside an agent's time interval τ_j , and $r_{h,j} = \log(P_{lit}(t)/P_{lit}(t-\tau_j))$. τ_j is determined by random variables uniformly distributed in the interval $[\tau_{min}, \tau_{max}]$ at the start of the simulation independently for each agent. The first term of Eq. 1 represents a fundamental strategy: an agent expects a positive return when the market price is lower than the fundamental value, and vice versa. The second term of Eq. 1 represents a technical strategy: an agent expects a positive return when the historical market return is positive, and vice versa. After the expected log return $r_{e,j}(t)$ has been determined by the equation 1, an expected price $P_{o,j}(t)$ is determined as follows.

$$P_{o,j}(t) = P_{lit}(t) \cdot \exp(r_{e,j}(t)) \quad (2)$$

We modeled an order price by random variables of normal distributed in an average $P_{o,j}(t)$, a standard deviation P_σ , where P_σ is a constant. A minimum unit of a price change (tick size) is δP , we round off a fraction of less than δP .

Buy or sell is determined by a magnitude relationship between the expect price $P_{e,j}(t)$ and the order price $P_{o,j}(t)$, that is,

$$\begin{aligned} &\text{When } P_{e,j}(t) > P_{o,j}(t), \text{ the agent orders to buy one share.} \\ &\text{When } P_{e,j}(t) < P_{o,j}(t), \text{ the agent orders to sell one share.} \end{aligned}$$

The stylized traders order in the lit market with a probability $1 - d_s$, in the dark pool with HFT with a probability $0.5d_s$, and in the dark pool without HFT with a probability $0.5d_s$. An order of a stylized trader is canceled if the order is not executed within the t_c^{st} steps.

Large investor A large investor determine similarly its order price and direction to buy or sell as a stylized trader. However, it differs from the stylized trader in the way of ordering: Slicing of large orders. After the expectation, the large investor start to place orders in μ units with a possibility p and continues to place μ units' orders in k time steps. The order prices, the direction (buy or sell), and the selected market do not change in those k steps. The order volume of the large investor is $\mu \times k$ times larger than that of the stylized trader. In this study, the parameters are set as $(p, \mu, k) = (0.05, 5, 5)$. The large investor orders in the lit market with a probability $1 - d_L$, in the dark pool with HFT with a probability $0.5d_L$, and in the dark pool without HFT with a probability $0.5d_L$.

HFT agent (market maker, MM) In this model, we introduced a market maker (MM)[8], an agent with a market making strategy that is one of major trading strategies of HFT. HFT, although not strictly defined at present, has roughly the following characteristics. It is a trading strategy that aims for small profits by placing orders automatically by a computer program to utilize the price difference in a very short time such as milliseconds and microseconds. There are some kinds of the trading strategy of HFT, and a market-making strategy occupies a large percentage of them. A market-making strategy places both buying and selling orders near a middle price by high frequency. Although it can provide the liquidity of trading, it may distort price formation.

MM can place its order just after each stylized trader placed its order in every time steps. That is, MM has much higher speed of ordering than the stylized trader. The trading strategy of MM consists of the following steps. Firstly if there is MM's own order in the markets, MM cancels them. Secondly, MM places one unit of buying order at a lower price $P_{buy,MM}(t)$ and one unit of selling order at a higher price $P_{sell,MM}(t)$ simultaneously (Fig. 4).

$$P_{buy,MM}(t) = \begin{cases} P_{ba}(t) & (P_{fair,MM}(t) \geq P_{bb}(t) + 0.5P_f\theta) \\ P_{fair,MM}(t) - 0.5P_f\theta & (\text{others}) \\ P_{bb}(t) - P_f\theta & (P_{fair,MM}(t) \leq P_{ba}(t) - 0.5P_f\theta) \end{cases} \quad (3)$$

$$P_{sell,MM}(t) = \begin{cases} P_{ba}(t) + P_f\theta & (P_{fair,MM}(t) \geq P_{bb}(t) + 0.5P_f\theta) \\ P_{fair,MM}(t) + 0.5P_f\theta & (\text{others}) \\ P_{bb}(t) & (P_{fair,MM}(t) \leq P_{ba}(t) - 0.5P_f\theta) \end{cases} \quad (4)$$

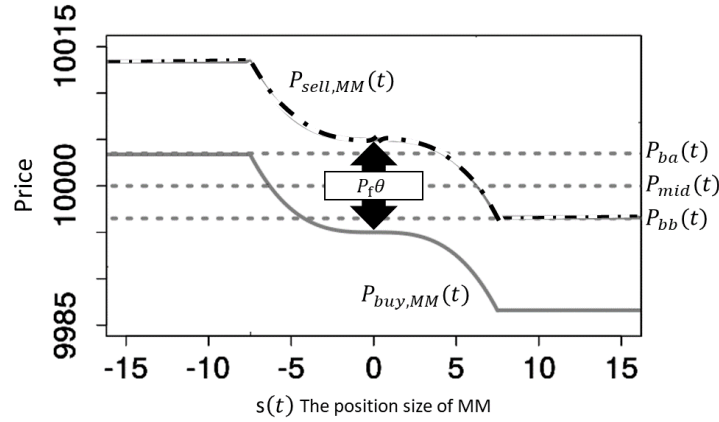


Fig. 4. Order price of MM

$P_{fair,MM}(t)$ is a fair price, a price level where MM can agree to buy or sell the asset. It is decided by the position size of MM's asset $s(t)$ and the mid price of the lit market $P_{mid}(t)$. θ is a spread rate (the ratio of difference between bid and ask prices in comparison to the fundamental price P_f).

$$P_{fair,MM}(t) = (1 - w_{MM} \cdot s(t)^3)P_{mid}(t) \quad (5)$$

The equations 3-4 means the position control of MM. That is, MM is willing to sell (buy) at a lower selling price $P_{sell,MM}(t)$ (higher buying price $P_{buy,MM}(t)$) when it has larger positive (negative) position $s(t)$ respectively. In this study, the value of w_{MM} is set to 2.0×10^{-6} .

MM usually uses the lit market for its orders. It however uses the HFT-accepted dark pool when the following two conditions are satisfied:

1. The selling price $P_{sell,MM}(t)$ (buying price $P_{buy,MM}(t)$) reaches the minimum price $P_{bb}(t)$ (the maximum price $P_{ba}(t)$) because of its large positive (negative) position respectively.
2. There are orders on the opposite side and MM's order can be transacted in the dark pool.

3 Simulation results

Using our model, we examined the influence on the market of the interaction among the dark pools, the large investor, and HFT.

3.1 Market efficiency and liquidity

We used the market inefficiency and liquidity as the indicators to measure the influence on the market. The market inefficiency M_{ie} is calculated based on the

difference between the fundamental price $P_f(t)$ and the lit market price $P_{lit}(t)$ [11]:

$$M_{ie} = \frac{1}{T} \sum_{t=t_s}^{t_s+T-1} \frac{|P_{lit}(t) - P_f(t)|}{P_f(t)}, \quad (6)$$

where t_s is the time step when the analysis period starts after the preparation period of simulation. T is the duration of the analysis period. In this study, $t_s = 500, T = 10,000$. If the divergence of $P_{lit}(t)$ from $P_f(t)$ is larger, the market is more inefficient.

The liquidity L_m of market $m = \{\text{lit, dark pool w/HFT, dark pool wo/HFT}\}$ is defined based on the transaction rate:

$$L_m = \frac{n_{trans,m}}{n_{order,m}}, \quad (7)$$

where $n_{trans,m}$ is a number of all transactions and $n_{order,m}$ is a number of all orders in market m during $t = (t_s, t_s + T - 1)$.

3.2 Parameter setting

In this study, values of the parameters are set as tables 1-4. These values were determined with reference to [11]. Under these parameter values, our model could replicate both long-term statistical characteristics (the fat-tail of return distribution and volatility clustering) and short-term market micro structures (transaction rates and cancel rates) of real financial markets.

Table 1. Parameters of markets

Start of analysis period	$t_s =$	500
Duration of analysis periods	$T =$	10,000
Tick size	$\delta P =$	1
Fundamental price	$P_f =$	10,000

Table 2. Parameters of stylized trader

Number of stylized traders	$n =$	100
Maximum weight of fundamental term	$w_{1,max} =$	1.0
Maximum weight of chartist term	$w_{2,max} =$	0.1
Maximum weight of noise term	$w_{3,max} =$	1.0
Minimum time interval of chart analysis	$\tau_{min} =$	100
Maximum time interval of chart analysis	$\tau_{max} =$	200
Standard deviation of noise term	$\sigma_\epsilon =$	0.001
Standard deviation of expected price	$P_\sigma =$	[0, 0.1]

Table 3. Parameters of large investor

Number of large investor	$n_{large} =$	1
Maximum weight of fundamental term	$w_{1,max} =$	1.0
Maximum weight of chartist term	$w_{2,max} =$	0.1
Maximum weight of noise term	$w_{3,max} =$	1.0
Time interval of chart analysis	$\tau_j =$	150
Standard deviation of noise term	$\sigma_\epsilon =$	0.001
Standard deviation of expected price	$P_\sigma =$	[0, 0.1]
Probability of slicing start	$p =$	0.05
Order size of each order	$\mu =$	5
Number of slicing	$k =$	5

Table 4. Parameters of market maker

Number of market maker	$n_{MM} =$	1
Spread rate	$\theta =$	0.001
Coefficient of position control	$w_{MM} =$	0.000002
Initial position	$s(0) =$	0
Duration of holding orders	$t_C =$	1

3.3 Market efficiency results

First, we examined the influence on the market efficiency of the dark pool without HFT. Figure 5 shows the relationship between the market inefficiency M_{ie} , the probability of dark pool orders by the stylized traders d_S , and the probability of dark pool orders by the large investor d_L . The more the stylized traders use the dark pool (the larger d_S), the less efficient the market is (the larger M_{ie}). On the contrary, the more the large investor uses the dark pool (the larger d_L), the more efficient the market is (the smaller M_{ie}), as Figure 6 shows. Next, we also examined the change of market efficiency with HFT. The market efficiency with HFT is larger than that without HFT.

We considered the mechanism of those two results is as follows:

1. When too many stylized traders use the dark pool (d_s is too large), the number and volume of orders in the lit market become too small. Then the price changes caused by one order become large and the lit market becomes unstable. That is, the market price $P_{lit}(t)$ tends to derive from the fundamental price $P_f(t)$ and the market inefficiency M_{ie} becomes larger.
2. When the large investor uses the lit market, its large order at a certain price disturbs the free price movement and sticks the market price $P_{lit}(t)$ away from the fundamental price $P_f(t)$. Thus, the large investor uses the dark pool (d_L is larger), the market price can return to the fundamental price level more easily and the market becomes more efficient (M_{ie} becomes smaller).
3. When HFT exists in a lit market, there are both selling and buying orders around the mid price constantly. Those orders block the market price's deviation from the fundamental price.

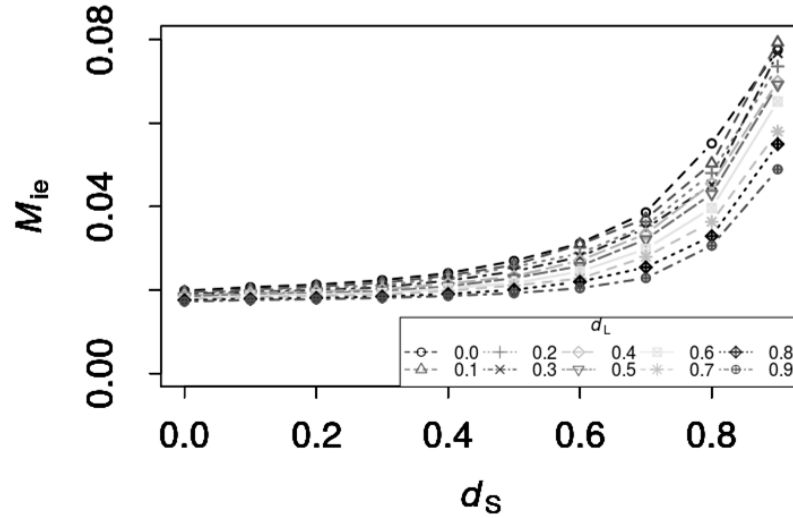


Fig. 5. Market inefficiency and the probability of dark pool orders by the stylized traders d_S

3.4 Market liquidity results

In this study, HFT trader is the market maker (MM). MM provides many orders to the markets and makes it easier to match orders. Thus, the liquidity (transaction rate) of the dark pool with HFT $L_{dark\ pool\ w/HFT}$ tends to be larger than the liquidity of the dark pool without HFT $L_{dark\ pool\ wo/HFT}$. We examined the change of the difference between those liquidity ΔL when the percentage of using the dark pools by the stylized traders d_S changed.

$$\Delta L = L_{dark\ pool\ w/HFT} - L_{dark\ pool\ wo/HFT} \quad (8)$$

Figure 7 shows the relationship between ΔL and d_S . The more stylized traders use the dark pools, the smaller the difference becomes. That is, the stylized trader's use of the dark pools reduced MM's contribution to the market liquidity.

We also examined the percentages of MM's use of the dark pool in changing the percentages of the stylized traders' use of the dark pools d_S . The results shows that the more the stylized traders use the dark pools, the less MM uses the dark pool. The decrease of MM's use of the dark pool reduces the liquidity advantage of the HFT-accepted dark pool in comparison with the HFT-prohibited dark pool.

The mechanism of those results is considered as follows. MM usually uses the lit market. It however uses the HFT accepted dark pool when it has a large positive or negative amount of assets and there are orders on the opposite

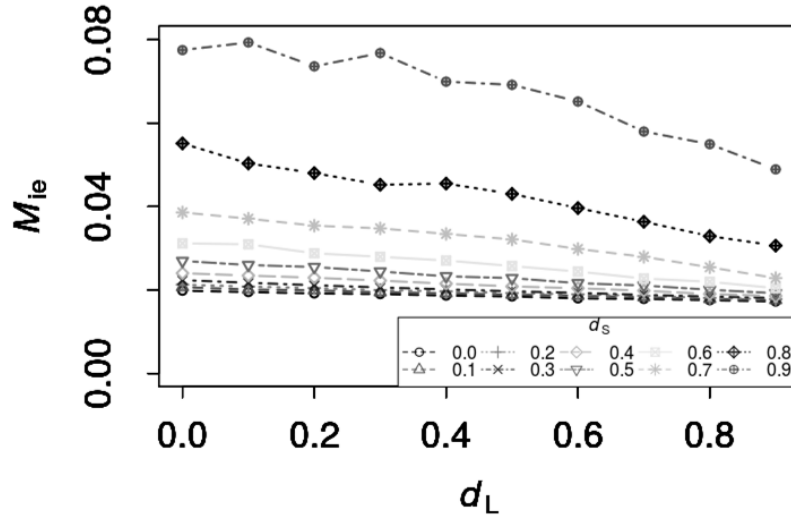


Fig. 6. Market inefficiency and the probability of dark pool orders by the large investor d_L

side in the dark pool. The simulation data shows that the rate at which the first condition (the large amount of assets) was satisfied was almost constant at every utilization rate of the stylized traders d_S . On the contrary, the rate at which the second condition (the existence of opposite orders in the dark pool) is satisfied decreases as d_S increases. When the stylized traders uses the dark pools frequently (d_S is larger), the orderbooks of the dark pools become similar to the orderbook of the lit market. Thus, the unbalance of orders and the absence of opposite orders tend to appear both in the lit market and the dark pools. That is, the second condition is not satisfied more frequently.

4 Conclusions

In this research, we expanded an artificial market model on the bases of the model of [11, 8] including the lit market, the dark pools, the large investor, and the market maker (HFT). Using the model, we investigated their influences on the market efficiency and liquidity.

We found that too much usage of the dark pool by the stylized traders (normal traders) would lead to the inefficiency of the market (the large deviation from the fundamental price). This result is similar to the preceding study's results[11]. On the contrary, the large investor's usage of the dark pool made the market more efficient. The market maker (HFT) improved the market liquidity (the transaction rate of orders). However, the increasing usage of the dark pool

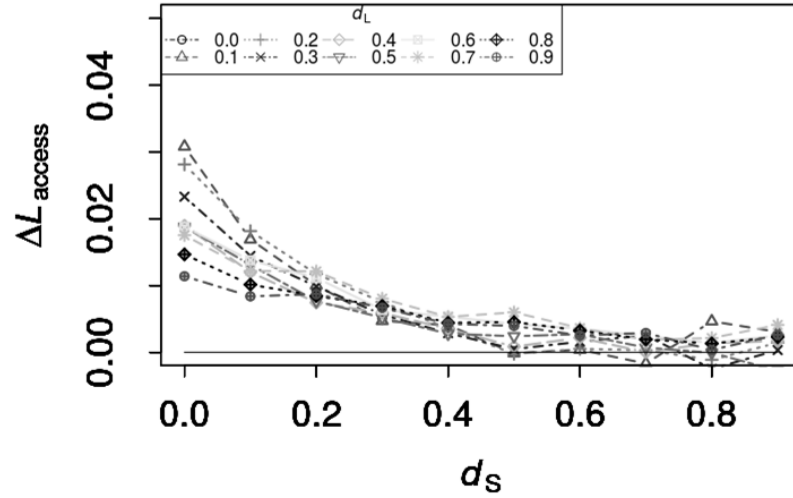


Fig. 7. Difference of liquidity and the probability of dark pool orders by the stylized traders d_S

by the stylized traders reduced the improvement rate of the market liquidity by HFT.

These results suggested that dark pools may improve the market efficiency if their usage rate were under some threshold. Especially, It is desirable that the main users of the dark pool are large investors. A certain kind of HFT such as the market making strategy may provide the market liquidity if their usage rate were under some threshold.

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References

1. Bogard, V.: High-frequency trading: An important conversation. Tech. rep., Tabb Forum (2014), <http://tabbforum.com/opinions/high-frequency-trading-an-important-conversation>
2. Chen, S.H., Chang, C.L., Du, Y.R.: Agent-based economic models and econometrics. Knowledge Eng. Review **27**(2), 187–219 (2012)
3. Chiarella, C., Iori, G., Perell, J.: The impact of heterogeneous trading rules on the limit order book and order flows **33**, 525–537 (03 2009)

4. European Commission and others: Public consultation review of the markets in financial instruments directive (mifid). Tech. Rep. 8, Consultation Report (2010)
5. Friedman, D.: The double auction market institution: A survey. *The Double Auction Market: Institutions, Theories, and Evidence* pp. 3–25 (1993)
6. Friedman, D.: Tse equity market summary after arrowhead launch (2011), http://www.tse.or.jp/english/news/47/b7gje6000000fck3-att/20110225_a.pdf
7. Johnson, B.: *Algorithmic Trading and DMA: An introduction to direct access trading strategies*. Myeloma Press (2010)
8. Kusada, Y., Mizuta, T., Hakawa, S., Izumi, K.: Impacts of position-based market makers on markets' shares of trading volumes: An artificial market approach. In: *Social Modeling and Simulations + Econophysics Colloquium 2014* (2014)
9. Lebaron, B.: Agent-based computational finance. In: Tesfatsion, L., Judd, K.L. (eds.) *Handbook of Computational Economics*, vol. 2, chap. 24, pp. 1187–1233. Elsevier, 1 edn. (2006)
10. McCrank, J.: Dark markets may be more harmful than high-frequency trading. Tech. rep., Reuters (2014)
11. Mizuta, T., Hayakawa, S., Izumi, K., Yoshimura, S.: Simulation study on effects of tick size difference in stock markets competition. In: *International Workshop on Agent-based Approaches in Economic and Social Complex Systems 2013* (2013)
12. Mizuta, T., Izumi, K., Yoshimura, S.: Price variation limits and financial market bubbles: Artificial market simulations with agents, learning process. In: *2013 IEEE Symposium Series on Computational Intelligence. Computational Intelligence for Financial Engineering Economics (CIFEr)* (2013)
13. Mo, S.Y.K., Yan, M.P.S.Y.: A study of dark pool trading using an agent-based model. In: *2013 IEEE Symposium Series on Computational Intelligence. Computational Intelligence for Financial Engineering Economics (CIFEr)* (2013)
14. Mostowfi, S., Bogard, V.: Us equity market structure: Q4-2015 tabb equity digest. Tech. rep., Tabb Group Report (2016)
15. Securities and Exchange Commission (SEC): Concept release on equity market structure. *Federal Register* **75**(13), 3594–3614 (2010)
16. Urrutia, J.P.: Progress in the mifid review: Stocktaking at the end of the irish presidency of the european council (2013), http://www.itg.com/marketing/ITG_Blotter_JP_Urrutia_MiFID_Review_20130607.pdf
17. Wah, E., Lahaie, S., Pennock, D.M.: An empirical game-theoretic analysis of price discovery in prediction markets. In: *Proceedings of the Twenty-Fifth International Joint Conference on Artificial Intelligence*. pp. 510–516. IJCAI'16, AAAI Press (2016)
18. Westerhoff, F.: The use of agent-based financial market models to test the effectiveness of regulatory policies. *Journal of Economics and Statistics (Jahrbuecher fuer Nationaloekonomie und Statistik)* **228**(2-3), 195–227 (2008)