Estimating Hidden Relation in Economic Activity

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Abstract. Due to the economic globalization, the networks of the economic relationship becomes more and more complex. There must be many unwritten records of economic activities. If we can reveal such hidden relationship, it may help us understand economic relations more deeply. In this study, we pick up an input-output table which represents a quantitative economic relation between different branches of industries. We propose a new approach to analyze the input-output table. We introduce two eigen vector based indices from the economic network which is derived from the input-output table. By introducing an imaginary node into the network so as not to change the current situation, we can obtain the hidden relation. The newly added links of the imaginary node represent the relation between the external factor and internal nodes. We demonstrate that the relation between industrial sectors and a government can be obtained from this method.

1 Introduction

Globalization has made the world economy more complex than ever. Nowadays, an economic network spreads over the world. Huge amount of activities are done in a complex network [**?**].

An input-output table represents a quantitative economic relation between different branches of national or international industries. The input-output table is a very old concept to capture the economic activities [2]. The Input-output table is constructed in various countries such as United States or European countries. Fig. 1 shows the basic structure of an input-output table.

Input-output tables have been used for long time [3]. However, this paper proposes a new approach to analyze the input-output table. We would like to estimate the effect of the external factors behind this table. For example, this table only shows the relationship between industrial sectors, but each industrial sector has also a relation with government. Or, sometimes, a large-scale natural disaster may impact on the whole economy. We will reveal these effects by introducing imaginary node (= sector) in the economic network which is represented by input-output tables. This is an inverse problem, therefore, some optimization method is needed. We solve this by a genetic algorithm.

		Demand sector
		Intermediate demand
Supply sector		1 2 3 ⋯ N
Intermediate input	1	
	2	
	3	
	⋮	
	N	

Fig. 1. The structure of an input output table

2 How to Find Influential Nodes or Vulnerable Nodes in the Economic Network

This study try to analyze the traditional input-output tables, but in a different way — from the viewpoint of a complex network analysis. Because the research of the complex network reveals the various interesting finding, we can evaluate a complex network by analyzing its graph structure. We can evaluate a complex network by manipulating the graph matrix [4].

2.1 Eigen Vector Centrality Based Indices

Here, in order to consider the influence of the node, we can imagine the following situation. If something is spread through a network, we can say the nodes are infected by something. The way of spread is based on the connection, i.e., the adjacency matrix. We define the weighted adjacency matrix as M. The probability of the infection at time t is $p(t)$. $p(t)$ can be written as the following equation[5].

$$p(t) = M^{t-1} p(0) \tag{1}$$

We can write the accumulative spread probability:

$$\begin{aligned} AI &= \sum_{t=1}^{\infty} p(t) \qquad (2) \\ &= (I + M + M^2 + \cdots + M^t + \cdots)p(0) \\ &\approx (I - M)^{-1} p(0) \propto (I - \alpha A)^{-1} \boldsymbol{e} \end{aligned}$$

This is called “amplification index” (AI).

We can think another type of index by tracing backward:

$$VI = (I + M^T + {M^T}^2 + \cdots + {M^T}^t + \cdots)p(0) \tag{3}$$

This is called “vulnerability index” (VI).

2.2 Impact Rank and Susceptibility Rank

We have more two indices from the different point of view. These two indices are based on LeaderRank [6][7]. LeaderRank is a measure of importance of the nodes. This index is a variation PageRank[8]. In order to calculate PageRank, we usually need a random walk through the network. LeaderRank can calculate PageRank quickly by introducing "ground node" which is a imaginary node connected to all nodes in the network. Fig. 2 shows the concept of the ground node.

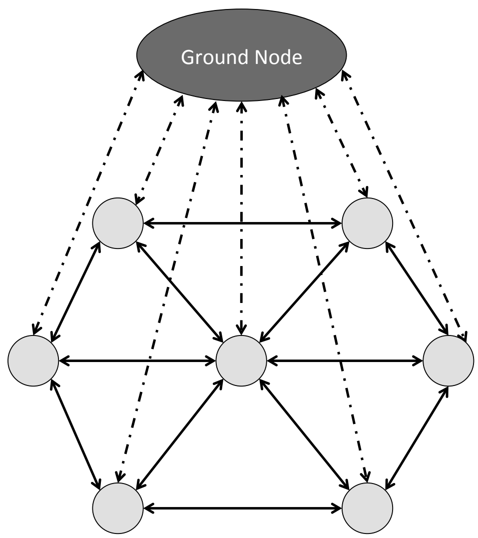

Fig. 2. Relation between ordinary nodes and a ground node

In this algorithm, the score of a node is calculated as following formula:

$$s_i(t+1) = \sum_{j=1}^{N+1} \frac{w_{ji}}{\sum_{l=1}^{N+1} w_{jl}} s_j(t) \tag{4}$$

If the steady state is achieved, Eq. (4) can be written as follow:

$$s_i(t_\infty) = \sum_{j=1}^{N+1} \frac{w_{gi}}{\sum_{l=1}^{N+1} w_{gl}} s_g(t_\infty) \tag{5}$$

To measure the impact of a sector to other sectors, we modify the Eq. (5):

$$s_i(t+1) = \sum_{j=1}^{N+1} \frac{w_{ji}}{k_j^{out}} s_j(t) \tag{6}$$

where k_j^{out} is the total out-degree of sector j. w_{ji} is an amount of transaction flow from sector j to sector i. We call this an impact rank. The impact rank is very similar to Amplification Index. To measure the susceptibility of a sector from other sectors, we modify Eq. (6):

$$s_i(t+1) = \sum_{j=1}^{N+1} \frac{w_{ij}}{k_j^{in}} s_j(t) \tag{7}$$

where k_j^{in} is the total in-degree of sector j. w_{ij} is an amount of transaction flow from sector i to sector j. We call this a susceptibility rank. The susceptibility rank is very similar to Vulnerability Index.

We interpret that a ground node is a source of external effect on a network. The weight of the link between ground node and other nodes in the network can be thought as the influence to/from the external effect.

3 Analysis of Japanese Input-Output Table

In this study, we analyze the Japanese economy using the input-output tables of Japan. In Japan, Ministry of Internal Affairs and Communications conducts to create input-output tables every five years [9]. We use the data compiled in 2005. There are several cases of input-output tables — original 520 sectors are summarized into, 190, 108, and 34 groups. We use 108 summarized version of the input-output table. Fig. 3 shows the flow in the 108-sized input-output table.

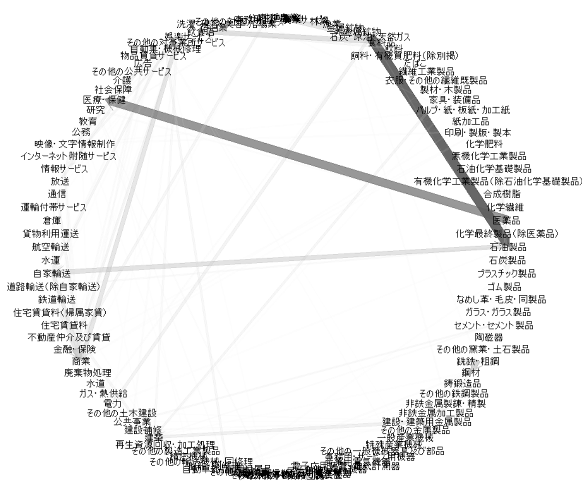

Fig. 3. Monetary flow in Industrial Network of Japan

Table 1 and Table 2 show the list of nodes sorted by the indices introduced in section 2. Table 1 shows the top ten influential sectors in the Japanese input-output network and Table 2 shows the top ten susceptible sectors in the Japanese input-output network.

We use these two indices as references of the following analysis.

Table 1. Top ten sectors of Japanese Economy in 2005 by Amplification Index (AI)

Rank	Sector Name
1	Motor vehicle parts and accessories
2	Steel products
3	Plastic products
4	Research
5	Non-ferrous metal products
6	Electrical devices and parts
7	Pig iron and crude steel
8	Other business services
9	Cast and forged steel products
10	Finance and insurance

Table 2. Top ten sectors of Japanese Economy in 2005 by Vulnerability Index (VI)

Rank	Sector Name
1	Motor vehicle parts and accessories
2	Passenger motor cars
3	Other cars
4	Repair of motor vehicles and machine
5	Self-transport by private cars
6	Commerce
7	Goods rental and leasing services
8	Other transportation equipment
9	Road transport
10	Electricity

4 Estimation of Hidden Relationship between Economic Sector and Goverment in Japanese Economy

4.1 Government as an Ground Node

In this section, we put a ground node into the industrial network. The weights between normal nodes (= sectors) are constant values determined from the input-output tables. On the other hand, we don't know any information about the weights between normal nodes and the ground node.

If we can adjust the weights between normal nodes and the ground node so as not to interfere with a ranking of nodes, we might say that we can introduce an external factor into the network. In order to do that, we will reproduce the same ranking order of Impact Rank as Amplification Index, and also reproduce the same ranking order of Susceptibility Rank as Vulnerability Index.

In economic network, a government is a candidate of the substance of the ground node because the government has monetary connections to all industrial sectors with tax (= in-degree), or grants (= out degree).

4.2 How to Find the Weight Configuration

To find such configuration of weights, we adopt the real coded Conditional Genetic Algorithm (rc-CGA) [10]. The genetic algorithm with conditional genetic operators (CGA) is a variant of GA motivated by the idea of selective breeding in evolutionary biology. In the CGA model, crossover and mutation behaviors are performed by difference-degree between chromosomes, and thus the probabilities of crossover and mutation are adjustable in optimization process. The following is the procedure of the genetic algorithm:

1. Initialize: to create a population. Chromosome consists of the array of link weights between the ground node and other normal nodes. Length of the chromosome = 2 * 108. Population size = 500.
2. Evaluation: to evaluate individuals by fitness value. Fitness value is calculated as the difference between two index (e.g. between Impact Rank and Amplification Index, or between Susceptibility Rank and Vulnerability Index).
3. Selection: to select a parents for crossover. Tournament selection is used. Tournament size = 10.
4. Crossover: to create offsprings. If the distance between two parents is larger than threshold, BLX-0.5 is used as crossover.
5. Mutation: to change chromosome. Non-Uniform mutation is used. In this case, the mutation distance decreases as generation passes.
6. If termination condition is not satisfied, update the distance parameter. Then go to 2.
7. Solution Set

4.3 Experimental Results

Fig. 4 shows the progress of evolution of rc-CGA. The fitness of the best chromosome seems to be saturated around 3,000 generation.

Table 3 shows the top ten sectors ranked by Impact Rank obtained from the best chromosome. Comparing this result with the result of Amplification Index (Table 1), rc-CGA can reproduce the ranking well.

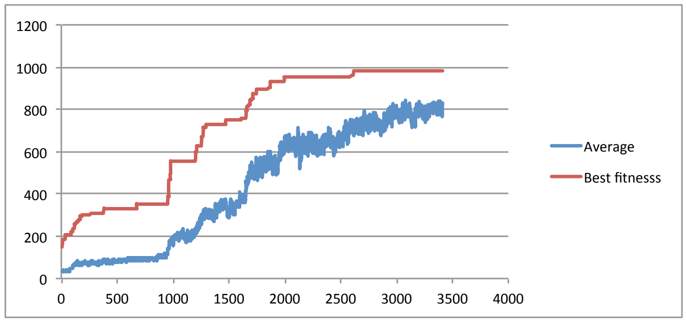

Fig. 4. Evolution of the best and average fitness values by comparing 108 sectors

Table 3. Top ten sectors of Japanese Economy in 2005 by Impact Rank (IR) generated by GA (Fitness type 1: difference between orders of all sectors)

Rank	Sector Name
1	Motor vehicle parts and accessories
2	Steel products
3	Plastic products
4	Research
5	Non-ferrous metal products
6	Electrical devices and parts
7	Pig iron and crude steel
8	Other business services
9	Cast and forged steel products
10	Finance and insurance

Table 4 shows the top ten sectors ranked by Susceptibility Rank obtained from the best chromosome. Comparing this result with the result of Vulnerability Index (Table 2), rc-CGA cannot reproduce the good ranking.

From the above results, we can say that it is easier to adjust the ranking of Impact Rank to the ranking of Amplification Index than to adjust the ranking of Susceptibility Rank to the ranking of Vulnerability Index. This shows the difficulty of adjusting is differ from the direction of flow.

To overcome this difficulty, we change the fitness function. In this case, comparison of ranking between two indices is limited within top 20 sectors (c.f. all 108 sectors are compared in previous setting). Table 5 shows the ranking of Impact Rank and Table 6 show the ranking of Susceptibility Rank. In the case of Susceptibility Rank, better ranking are obtained.

5 Discussion

We are interested in the value of link weights between the ground node and other normal nodes. Table 7 shows the top ten sectors ranked by the amount

Table 4. Top ten sectors of Japanese Economy in 2005 by Susceptibility Rank (SR) generated by GA (Fitness type 1: difference between orders of all sectors)

Rank	Sector Name
1	Special indstrial machinery
2	Repair of motor vehicles and machine
3	Self-tranport by private cars
4	Electronic computing equipment
5	Medical service and health
6	Commerce
7	Goods rental and leasing services
8	General industrial machinery
9	Road transport
10	Electricity

Table 5. Top ten sectors of Japanese Economy in 2005 by Impact Rank (IR) generated by GA (Fitness type 1: difference between orders of top 20 sectors)

Rank	Sector Name
1	Motor vehicle parts and accessories
2	Steel products
3	Plastic products
4	Research
5	Non-ferrous metal products
6	Electrical devices and parts
7	Pig iron and crude steel
8	Other business services
9	Cast and forged steel products
10	Finance and insurance

of transactions to the ground node. Table 8 shows the top ten sectors ranked by the amount of transactions from the ground node. These two rankings differ from other rankings such as AI, VI, IR and SR.

Table 7 may represent major tax paying company and Table 8 may represent major protected company.

6 Conclusion

There must be many unwritten records of economic activities because the recent economic networks becomes more and more complex. In this study, we pick up an input-output table which represents a quantitative economic relation between different branches of industries. This paper proposes a new approach of analyze the table. From the table, we can estimate the effect of the external factors which is behind the scene. By introducing imaginary node into the network so

Table 6. Top ten sectors of Japanese Economy in 2005 by Susceptibility Rank (SR) generated by GA (Fitness type 1: difference between orders of top 20 sectors)

Rank	Sector Name
1	Medical service and health
2	Passenger motor cars
3	Commerce
4	Repair of motor vehicles and machine
5	Self-transport by private cars
6	Special industrial machinery
7	Goods rental and leasing services
8	Finance and insurance
9	Road transport
10	Electricity

Table 7. Top ten sectors ranked by the amount of transactions to the ground node

Rank	Sector Name	w_{ig}
1	Motor vehicle parts and accessories	1.05×10^{17}
2	Steel products	5.78×10^{11}
3	Other personal services	7.65×10^{9}
4	Reuse and recycling	2.24×10^{9}
5	Amusement and recreational services	2.18×10^{9}
6	Final chemical products, n.e.c.	2.02×10^{9}
7	Metallic ores	1.76×10^{9}
8	Other general machines	1.51×10^{9}
9	Rubber products	1.11×10^{9}
10	Medicaments	6.49×10^{8}

w_{ig}: weight (= the amount of transactions) from a target node to the ground node

as not to change the current situation, we can obtain the hidden relation. The newly added links of the imaginary node represents the relation between external factors and internal factors. We demonstrate that the relation between industrial sectors and a government can be obtained from this method. Genetic algorithm is the powerful tool to do this job. However, this method needs to improve, for example, fitness design and computation time. We plan to apply this method to other type of complex network in the real world.

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Table 8. Top ten sectors ranked by the amount of transactions from the ground node

Rank	Sector Name	w_{gi}
1	Special industrial machinery	6.27×10^{14}
2	Non-ferrous metals	3.69×10^{11}
3	Organic chemical products	8.38×10^{10}
4	Electronic computing equipment	2.70×10^{10}
5	Other transportation equipment	1.12×10^{10}
6	Other electrical equipment	3.92×10^{9}
7	Rubber products	3.60×10^{9}
8	Education	3.00×10^{9}
9	Office supplies	2.77×10^{9}
10	Industrial inorganic chemicals	1.03×10^{9}

w_{gi}: weight (= the amount of transactions) from the ground node to a target node

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