Multi-factor Productivity and Evolutionary Accounting in Presence of (Persistently) Heterogeneous Firms

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Introduction

- Robust evidence across sectors (industries) and countries[1] consistently finds large-scale reallocation of outputs and inputs across individual firms
- Thanks to the longitudinal firm-level data, one can analyze
  - the reallocation across individual firms within narrowly defined sector
  - the connection of this reallocation to the aggregate productivity growth (APG) of that sector
- However, the method adopted in the literature to investigate this connection, i.e. the decomposition of APG, is potentially problematic
- We propose one new method, which
  - to different degree, solves several problems of the method in the literature
  - more importantly, highlights one new perspective to investigate this connection

[1] See Baily et al. (1992); Baldwin and Rafiquzzaman (1995); Bartelsman and Doms (2000); Disney et al. (2003); Dosi (2005); Syverson (2011) and among many others for countries like USA, Canada, UK, France, Italy, Netherlands, etc.

Stylized facts on heterogeneity

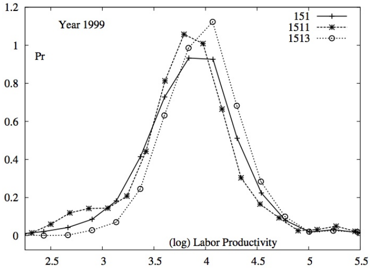
- **wide asymmetries in productivity across firms**
- equally wide heterogeneity in relative input intensities
- highly skewed distribution of efficiency, innovativeness and profitability indicators;
- different export status within the same industry

Stylized facts on heterogeneity

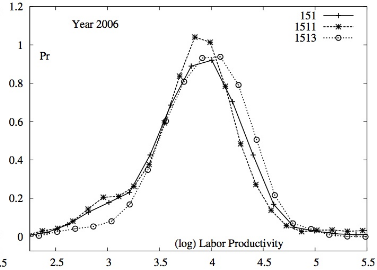
- high intertemporal persistence in the above properties
- high persistence of heterogeneity does NOT disappear when increasing the level of disaggregation
  - As Grilliches and Mairesse (1999) point out, *We [...] thought that one could reduce heterogeneity by going down from general mixtures as 'total manufacturing' to something more coherent, such as 'petroleum refining' or 'the manufacture of cement.' But something like Mandelbrot's fractal phenomenon seems to be at work here also:* ***observed variability-heterogeneity does not really decline as we cut our data finer and finer****. There is a sense in which different bakeries are just as much different from each others as the steel industry is from the machinery industry.*

Heterogeneity in performances and its persistence

Figure: Heterogeneous Productivity Distribution - Meat Products Industry in Italy



Year 1999



Year 2006

Decomposition of APG in previous studies

Before the decomposition, they **DEFINE** aggregate (industry) productivity at t as

$$\Pi^t := \sum_{i \in I^t} s_i^t \pi_i^t \tag{1}$$

with s_i^t representing the share of firm i in industry, π_i^t the individual productivity of firm i at time t and I^t the set including all firms within this industry at time t.

Thus the decomposition, following Grilliches and Regev (1995)[2], reads

$$APG := \Delta\Pi^t = \underbrace{\sum_{i \in C} \bar{s}_i \Delta\pi_i^t}_{within} + \underbrace{\sum_{i \in C} \bar{\pi}_i \Delta s_i^t}_{between} \tag{2}$$

where C denotes continuing firms, and for any variable or vector x^t at time t, operator Δ represents its change from $t-1$ to t i.e. $\Delta x^t := x^t - x^{t-1}$. The bar over a variable indicates the average of the variable over the base and end years, i.e. $\bar{x} := \frac{x^t + x^{t-1}}{2}$.

[2] Deviation term (Baily et al., 1992) can be neglected assuming there is no entering and exiting firm. See ▸ the method with enter and exiters according to Grilliches and Regev (1995) and one ▸ alternative method according to Baily et al. (1992) in the appendix.

Decomposition of APG in previous studies - problems

For aggregate productivity definition (1) $\Pi^t := \sum_i s_i^t \pi_i^t$

- Aggregate productivity level depends on the proxy for weight
- Accumulation of potential missing information

Firm	Labor	Output	Labor Productivity $:=$ Output$/$Labor
A	1	100	100
B	100	1	0.01

- According to (1), depending on the choice of weight, we have
  - (Output-weighted) $\Pi^t = \frac{100}{101} \cdot 100 + \frac{1}{101} \cdot 0.01 \approx 100$
  - (Labor-weighted) $\Pi_t = \frac{1}{101} \cdot 100 + \frac{100}{101} \cdot 0.01 = 1$

Decomposition of APG in previous studies - problems

For aggregate productivity definition (1) $\Pi^t := \sum_i s_i^t \pi_i^t$

- Aggregate productivity level depends on the proxy for weight
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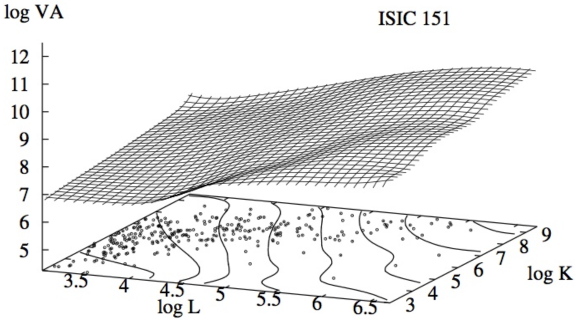
When it comes to multiple-input-output production activity,

- the productivity function π_i^t **CANNOT** grasp all the information of one firm's activity in one number since it maps one multiple-dimension vector to one real number
  - Genenrally used so-called total factor productivity (TFP): strong assumptions on production function (Hildenbrand, 1981; Dosi et al., 2016)[3]
- The more frequently we compute π_i^t, the more information we lose
  - to sum up individual production activity or individual productivity?
  - to measure one man's height, which ruler do you prefer? 2m or 20cm?

[3] As Hildenbrand (1981) points out "*short-run efficient production functions do not enjoy the well-known properties which are frequently assumed in production theory. For example, constant returns to scale never prevail, the production functions are never homothetic, and the elasticities of substitution are never constant. On the other hand, the competitive factor demand and product supply functions [. . .] will always have definite comparative static properties which cannot be derived from the standard theory of production*".

More fundamental problem - Something is missing

Figure: Heterogeneity in adopted techniques - Meat Products Industry in Italy, 2006



More fundamental problem - Something is missing

Reallocation of input and output involves two perspective:

- not only among firms - one firm with more output and/or less input compared to others for example
- but also within one firm - it changes the relative intensity of its inputs

However, the second perspective hasn't been taken into account in the decomposing APG literature

Based on Zonotopes(Hildenbrand, 1981; Dosi et al., 2016), here we

- argue that the heterogeneity in relative input intensities plays an important role in aggregate productivity and thus APG
- propose one new decomposition method accounting for the contribution of the change of this (input) heterogeneity to APG

Production activities in $\mathbb{R}^n$ space

Without loss of generality, during period t, the *ex post* technology of a production unit i is a vector

$$a_i^t = \left(\alpha_{i,1}^t, \cdots, \alpha_{i,(l-1)}^t, \alpha_{i,l}^t\right) \in \mathbb{R}_+^l \tag{3}$$

i.e. a **production activity** that produces $\alpha_{i,l}^t$ units of output by means of $\left(\alpha_{i,1}^t, \cdots, \alpha_{i,(l-1)}^t\right)$ units of input. (Koopmans, 1977; Hildenbrand, 1981)

- it also holds for the multi-output case

Production activities in $\mathbb{R}^3$ space - one toy example

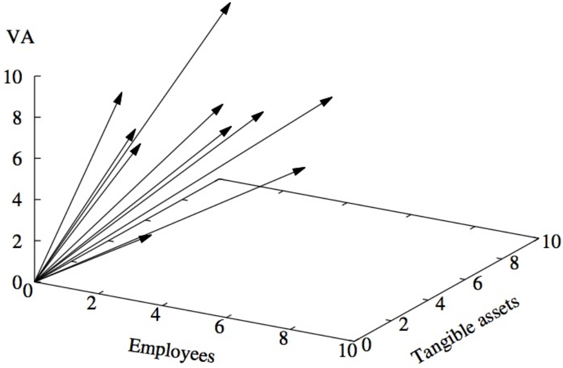


Figure: Production activities of some firms in $\mathbb{R}^3$ space

Production activities and Zonotopes

Given a_i^t, the *ex post* technology of a production unit i during period t,

- the short run production possibilities of an industry with N units during time t is a finite family of vectors $\{a_i^t\}_{1\leq i\leq N}$ of production activities
- Hildenbrand (1981) defines the **short run total production set** associated to them as the Zonotope[4]

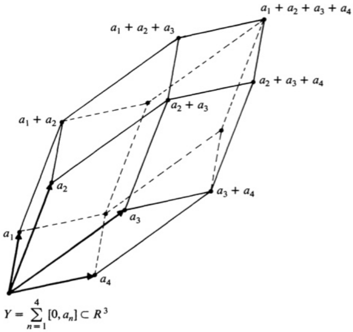
$$Z = \{z \in \mathbb{R}_+^l \,|\, z = \sum_{i=1}^{N} \phi_i a_i,\ \ 0 \leq \phi_i \leq 1\} \tag{4}$$

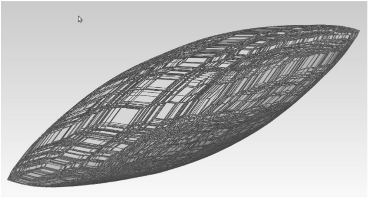
- Dosi et al. (2016) write the aggregate (industry) production activity d^t, i.e. the sum of individual firm production activity, as

$$d^t = (\beta_1^t, \cdots, \beta_{l-1}, \beta_l^t) = \left(\sum_{i=1}^{N} \alpha_{i,1}^t, \cdots, \sum_{i=1}^{N} \alpha_{i,(l-1)}^t, \sum_{i=1}^{N} \alpha_{i,l}^t\right) \in \mathbb{R}_+^l \tag{5}$$

[4]The generalization to any dimension of a Zonohedron that is a convex polyhedron where every face is a polygon with point symmetry or symmetry under rotations through 180°.

The Zonotopes - two toy examples

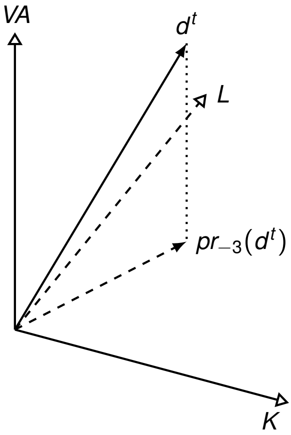




4-generator case (Hildenbrand, 1981) 300-generator case

Definition of productivity within Zonotope

The angle formed by a production activity vector with the space generated by all inputs expresses the productivity. And tangent of this angle can be a fine measure (Dosi et al., 2016).



Aggregate productivity at time t is defined as,

$$P^t := tg\left(\Theta_l(d^t)\right) = \frac{\beta_l^t}{||pr_{-l}\left(d^t\right)||} \tag{6}$$

and individual productivity follows

$$p_i^t := tg\left(\Theta_l(a_i^t)\right) = \frac{\alpha_{i,l}^t}{||pr_{-l}\left(a_i^t\right)||} \tag{7}$$

where

- $pr_{-l}(x)$ drops the l th element of vector x
- $||\cdot||$ represents the norm of one vector
- $\Theta_l(\cdot)$ denotes the angle measuring productivity (Dosi et al., 2016)

Aggregate productivity as average of individual productivity

It is easy to see

$$
\begin{aligned}
P^t &:= tg\left(\Theta_I(d^t)\right) \\
&= \frac{\beta_I^t}{||pr_{-I}(d^t)||} = \frac{\sum_{i\in I^t}\alpha_{i,I}^t}{||pr_{-I}(d^t)||} = \sum_{i\in I^t}\frac{||pr_{-I}\left(a_i^t\right)||}{||pr_{-I}(d^t)||}\cdot\frac{\alpha_{i,I}^t}{||pr_{-I}(a_i^t)||} \\
&= \sum_{i\in I^t} w_i^t p_i^t \qquad (8)
\end{aligned}
$$

where $w_i^t := \frac{||pr_{-I}\left(a_i^t\right)||}{||pr_{-I}\left(d^t\right)||}$ represents the input-based-weight defined as the relative length of individual input vector $pr_{-I}\left(a_i^t\right)$ over industry input vector $pr_{-I}\left(d^t\right)$.

Decomposition of APG within Zonotope framework

Following Grilliches and Regev (1995), given (8) and assuming there is no entering/exiting firm, we have decomposition of APG as

$$\text{APG} = \Delta P^t = \underbrace{\sum_{i \in C} \bar{w}_i \Delta p_i^t}_{\textit{Within}} + \underbrace{\sum_{i \in C} \bar{p}_i \Delta w_i^t}_{\textit{Between}^{as}} . \tag{9}$$

- Aggregate and individual productivities are defined in the same way
- $\sum_{i \in C} w_i^t$ is not necessarily equal to 1
- when there is only one input (labor), our method degenerates:
  - w_i^t degenerates to weight based on the level of this input and thus $\sum_{i \in C} w_i^t = 1$
  - P^t and p_i^t degenerate to productivity measure defined as ratio of output over input, i.e. labor productivity
- *as* in *Between*^{as} indicates **actual size** which will be discussed later

Decomposition of APG within Zonotope framework

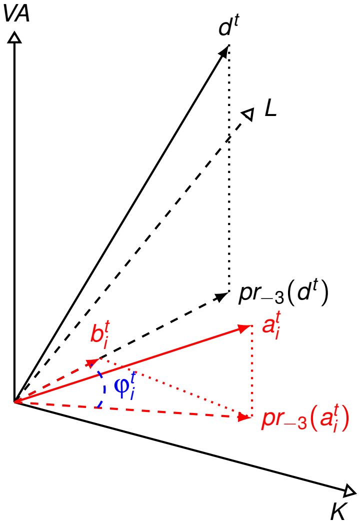
- In general, $\sum_{i \in C} w_i^t$ is not necessarily equal to 1
- When only 1 input, $\sum_{i \in C} w_i^t = 1$

What is behind?

Decomposition of APG within Zonotope framework

Directions of $pr_{-3}(a_i^t)$ and $pr_{-3}(d^t)$

- $\sum_{i \in C} w_i^t = 1$ when same directions
- $\sum_{i \in C} w_i^t \neq 1$ when different directions



Measuring the Input Heterogeneity

Decomposition of APG within Zonotope framework

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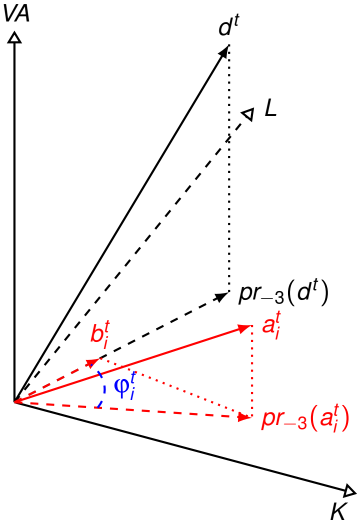
- $\sum_{i \in C} w_i^t \neq 1$ when different directions

Between $pr_{-3}(a_i^t)$ and $pr_{-3}(d^t)$ we define

- φ_i^t for the relative distance
- $||pr_{-3}(a_i^t)||$ for the individual **actual** size
- $||b_i^t||$ for the individual **contributing** size to $||pr_{-3}(d^t)||$



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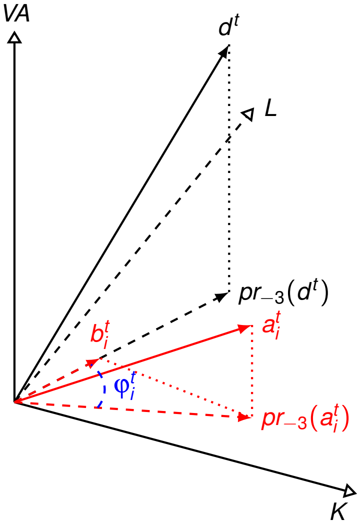
One Important Observation

$$||pr_{-3}(a_i^t)|| = \frac{||b_i^t||}{\cos\varphi_i^t}$$



Measuring the Input Heterogeneity

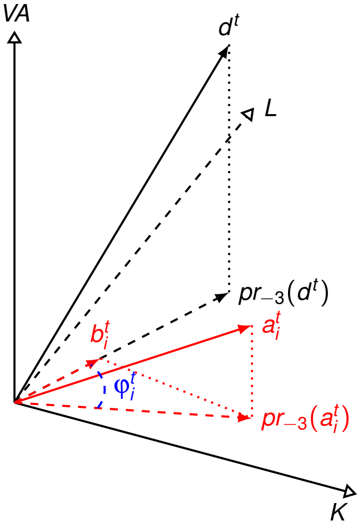
Decomposition of APG within Zonotope framework

One Important Observation

$$||pr_{-3}(a_i^t)|| = \frac{||b_i^t||}{\cos\varphi_i^t}$$

$$w_i^t = \frac{1}{||pr_{-3}(d^t)||} \cdot ||pr_{-3}(a_i^t)||$$
$$= \frac{1}{||pr_{-3}(d^t)||} \cdot \frac{||b_i^t||}{\cos\varphi_i^t}$$
$$= \underbrace{\frac{||b_i^t||}{||pr_{-3}(d^t)||}}_{s_i^t} \cdot \underbrace{\frac{1}{\cos\varphi_i^t}}_{h_i^t}$$

where
s_i^t denotes contributing weight
h_i^t denotes heterogeneity coefficient



Measuring the Input Heterogeneity

Decomposition of APG within Zonotope framework

We Just Learned

$$w_i^t = s_i^t \cdot h_i^t$$

Decomposition of APG within Zonotope framework

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$$w_i^t = s_i^t \cdot h_i^t$$

Thus it is easy to have

$$\Delta w_i^t = \bar{h}_i \Delta s_i^t + \bar{s}_i \Delta h_i^t$$

▸ Details

Decomposition of APG within Zonotope framework

We have

$$\Delta w_i^t = \bar{h}_i \Delta s_i^t + \bar{s}_i \Delta h_i^t$$

Recall Decomposition (9)

$$\text{APG} = \Delta P^t = \underbrace{\sum_{i \in C} \bar{w}_i \Delta p_i^t}_{Within} + \underbrace{\sum_{i \in C} \bar{p}_i \Delta w_i^t}_{Between^{as}}$$

Decomposition of APG within Zonotope framework

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Substituting the above left into the above right we have

$$\text{APG} = \Delta P^t = \underbrace{\sum_{i \in C} \bar{w}_i \Delta p_i^t}_{\textit{Within}} + \underbrace{\sum_{i \in C} \bar{p}_i \bar{h}_i \Delta s_i^t}_{\textit{Between}^{cs}} + \underbrace{\sum_{i \in C} \bar{p}_i \bar{s}_i \Delta h_i^t}_{\textit{Hetero}^{in}} \tag{10}$$

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Evolutionary accounting: APG is decomposed into 3 parts,

- *Within*, firm-level increase in productivity
- *Between*^{cs}, the reallocation of market share based on contributing size
- *Hetero*^{in}, change of heterogeneity in relative input intensities

Decomposition with Entering/Exiting Firms

Toy example

In $\mathbb{R}^3$ one Industry changes from Year 1 to Year 2 and satisfies

- Individual productivity does not change, i.e.
  - Individual output holds the same
  - Individual input size, i.e. $||pr_{-3}\left(a_i^t\right)||$, does not change
- Individual input-mixed becomes more diversified

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This indicates that in the input-plane $pr_{-3}\left(a_i^t\right)$ and $pr_{-3}\left(d^t\right)$ behave like

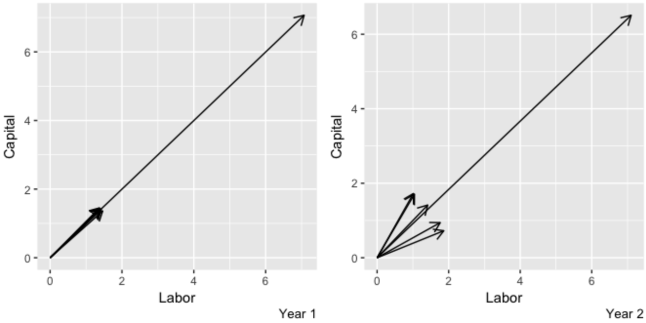

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Table: Making up one example with Firms' input-mixed more diversified from Year 1 to Year 2

	Year 1						Year 2					
	(1) L	(2) K	(3) Output	(4) length of input vector	(5) length of vector	(6) $tg(\cdot)$	(7) L	(8) K	(9) Output	(10) length of input vector	(11) length of vector	(12) $tg(\cdot)$
Firm 1	1.414	1.414	1.000	2.000	2.236	0.500	1.414	1.414	1.000	2.000	2.236	0.500
Firm 2	1.464	1.362	1.000	2.000	2.236	0.500	1.764	0.942	1.000	2.000	2.236	0.500
Firm 3	1.424	1.404	1.000	2.000	2.236	0.500	1.864	0.724	1.000	2.000	2.236	0.500
Firm 4	1.374	1.453	1.000	2.000	2.236	0.500	1.044	1.706	1.000	2.000	2.236	0.500
Firm 5	1.394	1.434	1.000	2.000	2.236	0.500	1.014	1.724	1.000	2.000	2.236	0.500
Industry	7.071	7.068	5.000	9.998	11.178	0.500*	7.101	6.510	5.000	9.634	10.854	0.519

* Notice that this 0.5, different from other 0.5 in this column, is not precisely 0.5 but by rounding.

Toy example

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It is easy to see that from year 1 to year 2

- Individual productivity does not change
- More diversified input-mixed
- Industry productivity increases from 0.500 to 0.519, with more precise APG = 0.01889587

Toy example

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APG is mostly driven by increasing heterogeneity

Applying proposed decomposition method, we have

$$\underbrace{0.01889587}_{APG} = \underbrace{0}_{Within} + \underbrace{(-0.000130873)}_{Between^{cs}} + \underbrace{0.01902674}_{Hetero^{in}}$$

▸ More Toy Examples

Toy example

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Firm 5	1.394	1.434	1.000	2.000	2.236	0.500	1.014	1.724	1.000	2.000	2.236	0.500
Industry	7.071	7.068	5.000	9.998	11.178	0.500*	7.101	6.510	5.000	9.634	10.854	0.519

* Notice that this 0.5, different from other 0.5 in this column, is not precisely 0.5 but by rounding.

Recall the decomposition in previous studies indicated as (2)

$$APG := \Delta\Pi^t = \underbrace{\sum_{i\in C} \bar{s}_i \Delta\pi_i^t}_{within} + \underbrace{\sum_{i\in C} \bar{\pi}_i \Delta s_i^t}_{between} \quad \text{where } \Pi^t := \sum_{i\in I^t} s_i^t \pi_i^t$$

and, with output as weight proxy, we have the decomposition as

$$APG = 0.5 - 0.5 = \underbrace{0}_{within} + \underbrace{0}_{between} .$$

Empirical evidence

Preparing the data sample for the empirical investigation,

- firm-level data from AMADEUS, a commercial database provided by Bureau van Dijk[5]
- yearly observations reported from 2004 to 2013
- selected industries (4-digit NACE) of Germany, France, and Italy
  ▸ List of Selected Industries
- production activity assumed in $\mathbb{R}^3$
  - proxy 3 inputs by numbers of employees, material cost, and fixed assets
  - proxy 1 output by turnover value
  - these variables are measured by thousand euro and deflated at 4-digit NACE level with year 2010 as benchmark year

[5] Our access (October 2015) contains balance sheets and income statements about over 21 Million European firms over the period 2004-2013

Empirical evidence - Comparing with Previous Studies

Table: Empirical Example - Comparing Decomposition Results among Different Methods

Productivity Proxy		TFP										$tg(\cdot)$			
Weight Proxy		Labor			Output			Length of Input Vector							
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
NACE	Ctry.	APG	*Enter* *-Exit*	*Within* *Btw*	APG	*Enter* *-Exit*	*Within* *Btw*	APG	*Enter* *-Exit*	*Within* $Btw.^{as}$	$Btw.^{cs}$ $Hetero^{in}$	APG	*Enter* *-Exit*	*Within* $Btw.^{as}$	$Btw.^{cs}$ $Hetero^{in}$
2630	FR	5.80	4.42 -0.4	4.73 -2.95	18.82	19.86 -1.11	1.48 -1.42	25.46	26.55 -0.43	1.67 -2.33	-2.84 0.51	0.02	1.14 -0.34	0.47 -1.26	-1.25 -0.01
2630	DE	7.05	44.33 -2.84	-11.81 -22.63	-295.59	68.46 -7.43	-160.58 -196.05	2.90	38.06 -10.09	13.5 -38.58	-42.01 3.44	-0.06	1.04 -0.45	-1.81 1.17	1.52 -0.35

- $\Pi^t := \sum_{i \in I^t} s_i^t \pi_i^t$
  where s_i^t is proxied by “labor” or “output” and pi_i^t by TFP
- $P^t = \sum_{i \in I^t} w_i^t p_i^t$
  where $w_i^t := \frac{||pr_{-l}(a_i^t)||}{||pr_{-l}(d^t)||}$ and $p_i^t := tg\left(\Theta_l(a_i^t)\right)$
- $\Pi^t := \sum_{i \in I^t} w_i^t \pi_i^t$
  where $w_i^t := \frac{||pr_{-l}(a_i^t)||}{||pr_{-l}(d^t)||}$ and pi_i^t is proxied TFP

Empirical evidence - some decomposition results

The main objective of this empirical investigation

- is to show change of heterogeneity does contribute to APG
- is **NOT** to study/compare the difference of this contribution across industry/country although it would be interesting to do so

Table: Decomposition of the Selected Industries

		From 2004 to 2007							From 2010 to 2013						
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
NACE	Ctry.	APG	Enter	Exit	Continue	*Within* *Btw.*^{as}	*Btw.*^{cs} *Hetero*^{in}	*Gini* Growth (%)	APG	Enter	Exit	Continue	*Within* *Btw.*^{as}	*Btw.*^{cs} *Hetero*^{in}	*Gini* Growth (%)
1081	DE	0.66	0.78	0.00	-0.12	1 -1.11	-1.18 0.07	143.71	0.20	0.42	1.16	0.94	-0.08 1.02	1.17 -0.15	-71.55
1081	FR	-0.53	1.16	0.00	-1.69	-0.5 -1.19	-1.16 -0.03	21.99	0.02	0.00	0.45	0.47	-0.04 0.51	0.61 -0.1	-14.80
1081	IT	-1.23	0.49	0.00	-1.72	0.17 -1.9	-2.22 0.32	352.95	0.78	0.00	0.95	1.73	-0.1 1.84	1.93 -0.09	-17.06

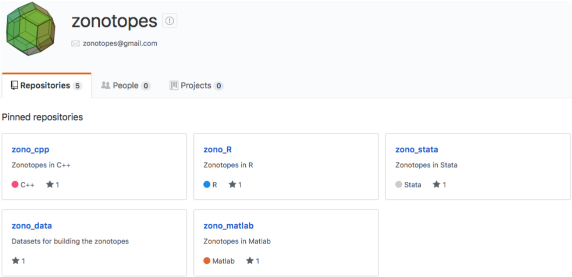
* Coloumns (7) and (14) report the percentage change of *Gini*-coefficient, introduced by Dosi et al. (2016) as one rigorous measure for heterogeneity.

Conclusion

- Many empirical literature has analyzed the relative importance between firm-level increase in productivity and the reallocation of market share, i.e. so-called "within" and "between" effects, to APG
- However, given the significant heterogeneity in relative input intensities, how change of this heterogeneity contributes to APG hasn't been investigated thoroughly.
- In this paper, we introduce one decomposition method which provides the possibility trying to account for this contribution together with the counterparts of the so-called "within" and "between" effects
- Empirical investigation indicates that the contribution from the change of firm-level heterogeneity is not trivial compared to the other components
  - the contribution itself is also heterogeneous across industry/country/time
- A straightforward step ahead involves
  - the study of dynamic characteristic of each industry/country/time
  - the explanation for the different contributions of change of firm-level heterogeneity across industry/country/time

Ongoing Zonotope project online

`https://github.com/zonotopes/`



- original codes for computing the heterogeneity measure, i.e. the *Gini* coefficient (Dosi et al., 2016) is based **C++**
- corresponding function/package for **R**, **Matlab**, and **STATA** are already available online
- the codes related to our proposed decomposition method will be online as soon as the working paper is published

THAT'S ALL
THANKS FOR YOUR ATTENTION

References

Aghion, P. and Howitt, P. (1992). A model of growth through creative destruction. *Econometrica*, 60:323–351.

Baily, M. N., Hulten, C., and Campbell, D. (1992). Productivity dynamics in manufacturing plants. *Brookings papers on economic activity (Microeconomics)*, 1992:187–249.

Baldwin, J. R. and Rafiquzzaman, M. (1995). Selection versus evolutionary adaptation: Learning and post-entry performance. *International Journal of Industrial Organization*, 13(4):501–522.

Bartelsman, E. J. and Doms, M. (2000). Understanding productivity: Lessons from longitudinal microdata. *Journal of Economic literature*, 38(3):569–594.

Caballero, R. J. and Hammour, M. L. (1994). The cleansing effect of recessions. *The American Economic Review*, 84(5):1350–1368.

Campbell, J. R. (1998). Entry, exit, embodied technology, and business cycles. *Review of economic dynamics*, 1(2):371–408.

Cooper, R., Haltiwanger, J., and Power, L. (1999). Machine replacement and the business cycle: Lumps and bumps. *The American Economic Review*, 89(4):921.

Dial, J. and Murphy, K. J. (1995). Incentives, downsizing, and value creation at general dynamics. *Journal of Financial Economics*, 37(3):261–314.

Disney, R., Haskel, J., and Heden, Y. (2003). Entry, exit and establishment survival in uk manufacturing. *The Journal of Industrial Economics*, 51(1):91–112.

Dosi, G. (2005). Statistical regularities in the evolution of industries: a guide through some evidence and challenges for the theory. Technical report, LEM working paper series.

Dosi, G., Grazzi, M., Marengo, L., and Settepanella, S. (2016). Production theory: Accounting for firm heterogeneity and technical change. *The Journal of Industrial Economics*, 64(4):875–907.

Ericson, R. and Pakes, A. (1992). An alternative theory of firm and industry dynamics. Technical report, Cowles Foundation for Research in Economics, Yale University.

References (cont.)

Foster, L., Haltiwanger, J. C., and Krizan, C. J. (2001). Aggregate productivity growth: lessons from microeconomic evidence. In *New developments in productivity analysis*, pages 303–372. University of Chicago Press.

Griliches, Z. and Mairesse, J. (1999). *Production functions: The search for identification*. Cambridge University Press: Cambridge.

Griliches, Z. and Regev, H. (1995). Productivity and firm turnover in israeli industry. *Journal of Econometrics*, 65(175):175–203.

Hildenbrand, W. (1981). Short-run production functions based on microdata. *Econometrica*, 49(5):1095–1125.

Hopenhayn, H. and Rogerson, R. (1993). Job turnover and policy evaluation: A general equilibrium analysis. *Journal of political Economy*, 101(5):915–938.

Hopenhayn, H. A. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica: Journal of the Econometric Society*, 60(5):1127–1150.

Jovanovic, B. (1982). Selection and the evolution of industry. *Econometrica: Journal of the Econometric Society*, pages 649–670.

Koopmans, T. C. (1977). 5 examples of production relations. *The microeconomic foundations of macroeconomics*, page 144.

Lambson, V. E. (1991). Industry evolution with sunk costs and uncertain market conditions. *International Journal of Industrial Organization*, 9(2):171–196.

Mansfield, E., Schwartz, M., and Wagner, S. (1981). Imitation costs and patents: an empirical study. *The Economic Journal*, 91(364):907–918.

Nabseth, L. and Ray, G. F. (1974). *The diffusion of new industrial processes: An international study*. Cambridge University Press.

Nelson, R. R. and Winter, S. G. (2009). *An evolutionary theory of economic change*. harvard university press.

References (cont.)

Pakes, A. and Schankerman, M. (1984). The rate of obsolescence of patents, research gestation lags, and the private rate of return to research resources. In *R&D, patents, and productivity*, pages 73–88. University of Chicago Press.

Roberts, K. and Weitzman, M. L. (1981). Funding criteria for research, development, and exploration projects. *Econometrica: Journal of the Econometric Society*, pages 1261–1288.

Syverson, C. (2011). What determines productivity? *Journal of Economic literature*, 49(2):326–365.

Theoretical underpinnings for firm-level heterogeneity

Literature suggests several models[6], using idiosyncratic factors as dominating determination, to explain firm-level heterogeneity

- Uncertainty
  - about new products or technologies (Roberts and Weitzman, 1981)
  - of experimentation (Jovanovic, 1982; Ericson and Pakes, 1992)
  - about future cost or demand conditions (Lambson, 1991)
- Different entrepreneurial and managerial ability (Dial and Murphy, 1995)
- Firm-specific location and disturbances

  (Hopenhayn, 1992; Hopenhayn and Rogerson, 1993; Campbell, 1998)
- Slow diffusion of information about technology and etc.

  (Nabseth and Ray, 1974; Mansfield et al., 1981; Pakes and Schankerman, 1984)
- Vintage of manager or organizational structure (Nelson and Winter, 2009)

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[6] They are closely related to the models emphasizing the role of creative destruction (Aghion and Howitt, 1992; Caballero and Hammour, 1994; Campbell, 1998; Cooper et al., 1999).

Decomposition of APG

The decomposition of APG, according to Baily et al. (1992), reads

$$\Delta\Pi^t = \underbrace{\sum_{i\in C} s_i^{t-1}\Delta\pi_i^t}_{within} + \underbrace{\sum_{i\in C}\left(\pi_i^{t-1}-\Pi^{t-1}\right)\Delta s_i^t}_{between} + \underbrace{\sum_{i\in C}\Delta\pi_i^t\Delta s_{i,t}}_{\text{cross term}}$$
$$+\underbrace{\sum_{i\in N} s_i^t\left(\pi_i^t-\Pi^{t-1}\right)}_{enter} - \underbrace{\sum_{i\in X} s_i^{t-1}\left(\pi_i^{t-1}-\Pi^{t-1}\right)}_{exiter} \tag{11}$$

where C denotes continuing firms, N denotes entering firms, X denotes exiting firms and for any variable or vector x^t at time t, operator Δ represents its change from $t-1$ to t i.e. $\Delta x^t := x^t - x^{t-1}$. And a bar over a variable indicates the average of the variable over the base and end years.

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Decomposition of APG

The decomposition of APG from Grilliches and Regev (1995), when enter and exiter are taken into account, reads

$$\Delta\Pi^t = \underbrace{\sum_{i\in C} \bar{s}_i \Delta\pi_i^t}_{within} + \underbrace{\sum_{i\in C} \left(\bar{\pi}_i - \bar{\Pi}\right) \Delta s_i^t}_{between} + \underbrace{\sum_{i\in N} s_i^t \left(\pi_i^t - \bar{\Pi}\right)}_{enter} - \underbrace{\sum_{i\in X} s_i^{t-1} \left(\pi_i^{t-1} - \bar{\Pi}\right)}_{exiter}$$

where C denotes continuing firms, N denotes entering firms, X denotes exiting firms and for any variable or vector x^t at time t, operator Δ represents its change from $t-1$ to t i.e. $\Delta x^t := x^t - x^{t-1}$. And a bar over a variable indicates the average of the variable over the base and end years.

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Decomposing the growth of individual product

For individual i at time t, we define one interested variable z_i^t as the product of these two known variables,

$$z_i^t := x_i^t \cdot y_i^t \tag{12}$$

and suppose that we are interested in decomposing the change of z_i^t from $t-1$ to t, i.e. Δz_i^t. Given that $x_i^t = x_i^{t-1} + \Delta x_i^t$ and $y_i^t = y_i^{t-1} + \Delta y_i^t$, it is easy to have

$$\begin{aligned}
\Delta z_i^t &= x_i^t y_i^t - x_i^{t-1} y_i^{t-1} \\
&= (x_i^{t-1} + \Delta x_i^t)(y_i^{t-1} + \Delta y_i^t) - x_i^{t-1} y_i^{t-1} \\
&= x_i^{t-1} \Delta y_i^t + y_i^{t-1} \Delta x_i^t + \Delta x_i^t \Delta y_i^t
\end{aligned} \tag{13}$$

Decomposing the growth of individual product

and if keeping rearranging the terms[7] on the right hand side of decomposition (13), we can have one alternative decomposition method as

$$\Delta z_i^t = \bar{x}_i \Delta y_i^t + \bar{y}_i \Delta x_i^t \ . \tag{14}$$

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[7] Indeed, we have both

$$x_i^{t-1}\Delta y_i^t + y_i^{t-1}\Delta x_i^t + \Delta x_i^t \Delta y_i^t = x_i^{t-1}\Delta y_i^t + (y_i^{t-1}\Delta x_i^t + \Delta x_i^t \Delta y_i^t) = x_i^{t-1}\Delta y_i^t + y_i^t \Delta x_i^t \quad \text{and}$$
$$x_i^{t-1}\Delta y_i^t + y_i^{t-1}\Delta x_i^t + \Delta x_i^t \Delta y_i^t = (x_i^{t-1}\Delta y_i^t + \Delta x_i^t \Delta y_i^t) + y_i^{t-1}\Delta x_i^t = x_i^t \Delta y_i^t + y_i^{t-1}\Delta x_i^t \ ,$$

and thus

$$\begin{aligned} x_i^{t-1}\Delta y_i^t + y_i^{t-1}\Delta x_i^t + \Delta x_i^t \Delta y_i^t &= \frac{1}{2}\left[(x_i^{t-1}\Delta y_i^t + y_i^t \Delta x_i^t) + (x_i^t \Delta y_i^t + y_i^{t-1}\Delta x_i^t)\right] \\ &= \frac{x_i^{t-1} + x_i^t}{2}\Delta y_i^t + \frac{y_i^{t-1} + y_i^t}{2}\Delta x_i^t \\ &= \bar{x}_i \Delta y_i^t + \bar{y}_i \Delta x_i^t \ . \end{aligned}$$

Decomposing APG with entering & exiting firms

Denote C, N, and X the sets of continues, entering, and exiting firms respectively[8]. Aggregate productivity firstly can be decomposed into

$$\Delta P^t = \left(\sum_{i \in C} \bar{w}_i \Delta p_i^t + \sum_{i \in C} \bar{p}_i \Delta w_i^t \right) + \sum_{i \in N} w_i^t p_i^t - \sum_{i \in X} w_i^{t-1} p_i^{t-1} \qquad (15)$$

where for **every** firm $i \in C$ Δw_i^t can be further decomposed and finally decomposition of APG follows,

$$\begin{aligned} \Delta P^t = & \underbrace{\sum_{i \in C} \bar{w}_i \Delta p_i^t}_{Within} + \underbrace{\sum_{i \in C} \bar{p}_i \bar{h}_i \Delta s_i^t}_{Between^{cs}} + \underbrace{\sum_{i \in C} \bar{p}_i \bar{s}_i \Delta h_i^t}_{Hetero^{in}} \\ & + \underbrace{\sum_{i \in N} w_i^t p_i^t - \sum_{i \in X} w_i^{t-1} p_i^{t-1}}_{Net-entry} . \end{aligned} \qquad (16)$$

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[8] To be specific, C denotes the firm set including all continuing firms, i.e. those are active in both $t-1$ and t, set N includes entering firms which are active in time t but not $t-1$ while set X includes exiting firms which are active in time $t-1$ but not t.

Toy examples - size effect of three different firms

Table: Toy Example 1 - Size Effect of Three Different Firms

		Year 1									Year 2								
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
		L	K	Output	length of input vector	$tg(\cdot)$	w_i	s_i	h_i	φ_i	L	K	Output	length of input vector	$tg(\cdot)$	w_i	s_i	h_i	φ_i
Case I	**Firm A**	1.732	1.000	1.000	2.000	0.500	0.352	0.331	1.063	0.347	17.321	10.000	10.000	20.000	0.500	0.854	0.851	1.003	0.083
	Firm B	1.414	1.414	1.000	2.000	0.500	0.352	0.351	1.004	0.085	1.414	1.414	1.000	2.000	0.500	0.085	0.084	1.016	0.179
	Firm C	0.518	1.932	1.000	2.000	0.500	0.352	0.319	1.105	0.439	0.518	1.932	1.000	2.000	0.500	0.085	0.065	1.311	0.703
	Industry	3.664	4.346	3.000	5.684	0.528	-	-	-	-	19.252	13.346	12.000	23.426	0.512	-	-	-	-
Case II	Firm A	1.732	1.000	1.000	2.000	0.500	0.352	0.331	1.063	0.347	1.732	1.000	1.000	2.000	0.500	0.084	0.081	1.041	0.282
	Firm B	1.414	1.414	1.000	2.000	0.500	0.352	0.351	1.004	0.085	14.142	14.142	10.000	20.000	0.500	0.845	0.845	1.000	0.020
	Firm C	0.518	1.932	1.000	2.000	0.500	0.352	0.319	1.105	0.439	0.518	1.932	1.000	2.000	0.500	0.084	0.074	1.142	0.503
	Industry	3.664	4.346	3.000	5.684	0.528	-	-	-	-	16.392	17.074	12.000	23.669	0.507	-	-	-	-
Case III	Firm A	1.732	1.000	1.000	2.000	0.500	0.352	0.331	1.063	0.347	1.732	1.000	1.000	2.000	0.500	0.086	0.067	1.288	0.681
	Firm B	1.414	1.414	1.000	2.000	0.500	0.352	0.351	1.004	0.085	1.414	1.414	1.000	2.000	0.500	0.086	0.078	1.095	0.420
	Firm C	0.518	1.932	1.000	2.000	0.500	0.352	0.319	1.105	0.439	5.176	19.319	10.000	20.000	0.500	0.859	0.855	1.005	0.104
	Industry	3.664	4.346	3.000	5.684	0.528	-	-	-	-	8.323	21.733	12.000	23.272	0.516	-	-	-	-

- One firm scales its input and output as 10 times
- Firm A, B, and C increasing the size as Case I, II, and III respectively
- Among three cases, AP decreases while the levels of this decreasing are different
- The same individual size effect contributes to APG differently according to how different its input-mixed is from industry input-mixed
- The more different, the less contribution on APG (case III)

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Toy examples - technology effect of three different firms

Table: Toy Example 2 - Technology Effect of Three Different Firms

		Year 1									Year 2								
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
		L	K	Output	length of input vector	$tg(\cdot)$	w_i	s_i	h_i	φ_i	L	K	Output	length of input vector	$tg(\cdot)$	w_i	s_i	h_i	φ_i
Case 1	*Firm A*	1.732	1.000	1.000	2.000	0.500	0.352	0.331	1.063	0.347	1.732	1.000	0.500	2.000	0.250	0.352	0.331	1.063	0.347
	Firm B	1.414	1.414	1.000	2.000	0.500	0.352	0.351	1.004	0.085	1.414	1.414	1.000	2.000	0.500	0.352	0.351	1.004	0.085
	Firm C	0.518	1.932	1.000	2.000	0.500	0.352	0.319	1.105	0.439	0.518	1.932	1.000	2.000	0.500	0.352	0.319	1.105	0.439
	Industry	3.664	4.346	3.000	5.684	0.528	-	-	-	-	3.664	4.346	2.500	5.684	0.440	-	-	-	-
Case 2	Firm A	1.732	1.000	1.000	2.000	0.500	0.352	0.331	1.063	0.347	1.732	1.000	1.000	2.000	0.500	0.352	0.331	1.063	0.347
	Firm B	1.414	1.414	1.000	2.000	0.500	0.352	0.351	1.004	0.085	1.414	1.414	0.500	2.000	0.250	0.352	0.351	1.004	0.085
	Firm C	0.518	1.932	1.000	2.000	0.500	0.352	0.319	1.105	0.439	0.518	1.932	1.000	2.000	0.500	0.352	0.319	1.105	0.439
	Industry	3.664	4.346	3.000	5.684	0.528	-	-	-	-	3.664	4.346	2.500	5.684	0.440	-	-	-	-
Case 3	Firm A	1.732	1.000	1.000	2.000	0.500	0.352	0.331	1.063	0.347	1.732	1.000	1.000	2.000	0.500	0.352	0.331	1.063	0.347
	Firm B	1.414	1.414	1.000	2.000	0.500	0.352	0.351	1.004	0.085	1.414	1.414	1.000	2.000	0.500	0.352	0.351	1.004	0.085
	Firm C	0.518	1.932	1.000	2.000	0.500	0.352	0.319	1.105	0.439	0.518	1.932	0.500	2.000	0.250	0.352	0.319	1.105	0.439
	Industry	3.664	4.346	3.000	5.684	0.528	-	-	-	-	3.664	4.346	2.500	5.684	0.440	-	-	-	-

- One firm becomes less productive
- Among Case 1, 2, and 3, Firm A, B, and C respectively decrease their productivity, i.e. output becomes half holding the inputs the same.
- AP decreases the same level, from 0.528 to 0.440 in all cases.
- The individual firm's contribution on AP does **NOT** depend on the size of φ_i, i.e. how close/faraway of individual input-mixed is from industry one

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Toy examples - entering effect of three different firms

Table: Toy Example 3 - Entering Effect of Three Different Firms

		Year 1									Year 2								
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
		L	K	Output	length of input vector	$tg(\cdot)$	w_i	s_i	h_i	φ_i	L	K	Output	length of input vector	$tg(\cdot)$	w_i	s_i	h_i	φ_i
Case I′	Firm A	1.732	1.000	1.000	2.000	0.500	0.352	0.331	1.063	0.347	1.732	1.000	1.000	2.000	0.500	0.085	0.085	1.003	0.083
	Firm B	1.414	1.414	1.000	2.000	0.500	0.352	0.351	1.004	0.085	1.414	1.414	1.000	2.000	0.500	0.085	0.084	1.016	0.179
	Firm C	0.518	1.932	1.000	2.000	0.500	0.352	0.319	1.105	0.439	0.518	1.932	1.000	2.000	0.500	0.085	0.065	1.311	0.703
	Firm A′	-	-	-	-	-	-	-	-	-	15.588	9.000	9.000	18.000	0.500	0.768	0.766	1.003	0.083
	Industry	3.664	4.346	3.000	5.684	0.528	-	-	-	-	19.252	13.346	12.000	23.426	0.512	-	-	-	-
Case II′	Firm A	1.732	1.000	1.000	2.000	0.500	0.352	0.331	1.063	0.347	1.732	1.000	1.000	2.000	0.500	0.084	0.081	1.041	0.282
	Firm B	1.414	1.414	1.000	2.000	0.500	0.352	0.351	1.004	0.085	1.414	1.414	1.000	2.000	0.500	0.084	0.084	1.000	0.020
	Firm C	0.518	1.932	1.000	2.000	0.500	0.352	0.319	1.105	0.439	0.518	1.932	1.000	2.000	0.500	0.084	0.074	1.142	0.503
	Firm B′	-	-	-	-	-	-	-	-	-	12.728	12.728	9.000	18.000	0.500	0.760	0.760	1.000	0.020
	Industry	3.664	4.346	3.000	5.684	0.528	-	-	-	-	16.392	17.074	12.000	23.669	0.507	-	-	-	-
Case III′	Firm A	1.732	1.000	1.000	2.000	0.500	0.352	0.331	1.063	0.347	1.732	1.000	1.000	2.000	0.500	0.086	0.067	1.288	0.681
	Firm B	1.414	1.414	1.000	2.000	0.500	0.352	0.351	1.004	0.085	1.414	1.414	1.000	2.000	0.500	0.086	0.078	1.095	0.420
	Firm C	0.518	1.932	1.000	2.000	0.500	0.352	0.319	1.105	0.439	0.518	1.932	1.000	2.000	0.500	0.086	0.085	1.005	0.104
	Firm C′	-	-	-	-	-	-	-	-	-	4.659	17.387	9.000	18.000	0.500	0.773	0.769	1.005	0.104
	Industry	3.664	4.346	3.000	5.684	0.528	-	-	-	-	8.323	21.733	12.000	23.272	0.516	-	-	-	-

- One firm, with the same productivity level as one incumbent firm, entering the industry at Year 2
- Among three cases, the entering firm uses the same input-mixed as Firm A, B, and C respectively
- Further assume the size this entering firm is 9 times as those of the incumbent firms
- It becomes interesting to compare the decomposition results between Toy Example 1 and 3

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List of selected industries

Table: List of Selected Industries

NACE	Name of Industry
1081	Manufacture of sugar
1091	Manufacture of prepared feeds for farm animals
1310	Preparation and spinning of textile fibres
1920	Manufacture of refined petroleum products
2011	Manufacture of industrial gases
2012	Manufacture of dyes and pigments
2013	Manufacture of other inorganic basic chemicals
2014	Manufacture of other organic basic chemicals
2015	Manufacture of fertilisers and nitrogen compounds
2016	Manufacture of plastics in primary forms
2017	Manufacture of synthetic rubber in primary forms
2211	Manufacture of rubber tyres and tubes; retreading and rebuilding of rubber tyres
2221	Manufacture of plastic plates, sheets, tubes and profiles
2351	Manufacture of cement
2352	Manufacture of lime and plaster
2451	Casting of iron
2452	Casting of steel
1011	Processing and preserving of meat
1105	Manufacture of beer
1411	Manufacture of leather clothes
1413	Manufacture of other outerwear
1520	Manufacture of footwear
2611	Manufacture of electronic components
2630	Manufacture of communication equipment
2640	Manufacture of consumer electronics

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Decomposing APG in general case - definitions & notations

During period t, production unit i, which is described by the vector

$$a_i^t = \left(\alpha_{i,1}^t, \cdots, \alpha_{i,m}^t, \alpha_{i,m+1}^t, \cdots \alpha_{i,m+n}^t\right) \in \mathbb{R}_+^{m+n} \tag{17}$$

produces $\left(\alpha_{i,m+1}^t, \cdots \alpha_{i,m+n}^t\right)$ units of n kinds of outputs by means of $\left(\alpha_{i,1}^t, \cdots, \alpha_{i,m}^t\right)$ units of m kinds of inputs. Thus the aggregate (industry) production activity d^t, as the sum of individual firm's activity, can be written as

$$\begin{aligned} d^t &= \left(\beta_1^t, \cdots, \beta_m^t, \beta_{m+1}^t, \cdots, \beta_{m+n}^t\right) \\ &= \left(\sum_{i \in I^t} \alpha_{i,1}^t, \cdots, \sum_{i \in I^t} \alpha_{i,m}^t, \sum_{i \in I^t} \alpha_{i,m+1}^t \cdots, \sum_{i \in I^t} \alpha_{i,m+n}^t\right) \in \mathbb{R}_+^{m+n}. \end{aligned} \tag{18}$$

where set I^t including all firms within one industry at time t.

Decomposing APG in general case - definitions & notations

Given a vector $v = (x_1, \cdots, x_m, x_{m+1}, \cdots, x_{m+n}) \in \mathbb{R}^{m+n}$, we denote two projection maps

$$pr_{in} : \mathbb{R}^{m+n} \to \mathbb{R}^m$$
$$(x_1, \cdots, x_m, x_{m+1}, \cdots, x_{m+n}) \to (x_1, \cdots, x_m)$$

and

$$pr_{out} : \mathbb{R}^{m+n} \to \mathbb{R}^n$$
$$(x_1, \cdots, x_m, x_{m+1}, \cdots, x_{m+n}) \to (x_{m+1}, \cdots, x_{m+n}) .$$

Decomposing APG in general case - definitions & notations

Thus AP and individual firm's productivity at time t can be extended as,

$$P^t := tg\left(\Theta(d^t)\right) = \frac{||pr_{out}\left(d^t\right)||}{||pr_{in}\left(d^t\right)||} \tag{19}$$

and

$$p_i^t := tg\left(\Theta(a_i^t)\right) = \frac{||pr_{out}\left(a_i^t\right)||}{||pr_{in}\left(a_i^t\right)||} \tag{20}$$

respectively where $\Theta(\cdot)$ is the counterpart of $\Theta_I(\cdot)$ in (6) and (7).

- we apply to multiple outputs the same principle already used for multiple inputs in definitions (6) and (7).
- between (19) and (20), we do not necessarily have $||pr_{out}\left(d^t\right)|| = \sum_{i \in I^t} ||pr_{out}\left(a_i^t\right)||$ except for there is only one output

Decomposing APG in general case

- Still define φ_i^t as the angle formed by the individual input vector and industry input vector, i.e. $pr_{in}\left(a_i^t\right)$ and $pr_{in}\left(d^t\right)$
- Newly define σ_i^t as the angle formed by the individual output vector and industry output vector, i.e. $pr_{out}\left(a_i^t\right)$ and $pr_{out}\left(d^t\right)$.

It is easy to have,

$$||pr_{out}\left(d^t\right)|| = \sum_{i\in I^t}\left(||pr_{out}\left(a_i^t\right)||\cos\sigma_i^t\right) \ . \tag{21}$$

As a result, AP, defined as (19), can be further written as the "weighted-average" of individual productivity p_i^t,

$$\begin{aligned} P^t = \frac{||pr_{out}\left(d^t\right)||}{||pr_{in}\left(d^t\right)||} &= \frac{\sum_{i\in I^t}\left(||pr_{out}\left(a_i^t\right)||\cos\sigma_i^t\right)}{||pr_{in}\left(d^t\right)||} \\ &= \sum_{i\in I^t}\left(\cos\sigma_i^t \frac{||pr_{in}\left(a_i^t\right)||}{||pr_{in}\left(d^t\right)||}\frac{||pr_{out}\left(a_i^t\right)||}{||pr_{in}\left(a_i^t\right)||}\right) \ . \end{aligned}$$

Decomposing APG in general case

Thus we get the natural decomposition of APG as

$$P^t = \sum_{i \in I^t} u_i^t p_i^t \tag{22}$$

where the "weight" coefficient

$$u_i^t := k_i^t \cdot w_i^t \tag{23}$$

is defined as the product of the output-based-homogeneity measure

$$k_i^t := \cos \sigma_i^t$$

and the input-based-weight

$$w_i^t := \frac{\left|\left|pr_{in}\left(a_i^t\right)\right|\right|}{\left|\left|pr_{in}\left(d^t\right)\right|\right|} .$$

- different from $h_i^t := \frac{1}{\cos \varphi_i^t}$, k_i^t is decreasing function of σ_i^t
- smaller σ_i^t indicates closer individual output $pr_{out}(a_i^t)$ is to industry output $pr_{out}(d^t)$, i.e. more output-based-homogeneity and bigger k_i^t

Denote C the set of continues firms, given (22), thus we have

$$APG = \sum_{i \in C} \bar{u}_i \Delta p_i^t + \sum_{i \in C} \bar{p}_i \Delta u_i^t \ . \tag{24}$$

According to (14), for every firm $i \in C$, we have

$$\begin{aligned} \Delta u_i^t &= \bar{k}_i \Delta w_i^t + \bar{w}_i \Delta k_i^t \\ &= \bar{k}_i \left(\bar{h}_i \Delta s_i^t + \bar{s}_i \Delta h_i^t \right) + \bar{w}_i \Delta k_i^t \ . \end{aligned} \tag{25}$$

Finally by substituting (25) into (24), we have decomposition of APG as

$$APG = \underbrace{\sum_{i \in C} \bar{u}_i \Delta p_i^t}_{Within} + \underbrace{\sum_{i \in C} \bar{p}_i \bar{k}_i \bar{h}_i \Delta s_i^t}_{Between^{cs}} + \underbrace{\sum_{i \in C} \bar{p}_i \bar{k}_i \bar{s}_i \Delta h_i^t}_{Hetero^{in}} + \underbrace{\sum_{i \in C} \bar{p}_i \bar{w}_i \Delta k_i^t}_{Homo^{out}} \tag{26}$$

where $Homo^{out}$ represents the contribution from the change of homogeneity of the output to the APG[9].

[9] Notice that when there is only one output, for all firm i over all time t, we have $k_i^t = \cos \sigma_i^t = 1$ and thus $\bar{k}_i = 1$ and $\Delta k_i^t = 0$. As a result, $Homo^{out}$ degenerates to 0.

Decomposing APG in general case - unbalanced case where $I^t \neq I^{t-1}$

We denote $C = I^t \cap I^{t-1}$, $N = I^t \setminus I^{t-1}$, and $X = I^{t-1} \setminus I^t$ the sets of continues, entering, and exiting firms respectively. Again according to (22) that AP can be written as the "weighted-average" of individual productivity p_i^t, thus we have

$$\Delta P^t = \sum_{i \in C} \bar{u}_i \Delta p_i^t + \sum_{i \in C} \bar{p}_i \Delta u_i^t + \sum_{i \in N} u_i^t p_i^t - \sum_{i \in X} u_i^{t-1} p_i^{t-1}$$

where for **every** firm $i \in C$ we continue the decomposition of Δu_i^t according to (25) and finally decomposition of APG follows,

$$\Delta P^t = \underbrace{\sum_{i \in C} \bar{u}_i \Delta p_i^t}_{Within} + \underbrace{\sum_{i \in C} \bar{p}_i \bar{k}_i \bar{h}_i \Delta s_i^t}_{Between^{cs}} + \underbrace{\sum_{i \in C} \bar{p}_i \bar{k}_i \bar{s}_i \Delta h_i^t}_{Hetero^{in}} + \underbrace{\sum_{i \in C} \bar{p}_i \bar{w}_i \Delta k_i^t}_{Homo^{out}} + \underbrace{\sum_{i \in N} u_i^t p_i^t - \sum_{i \in X} u_i^{t-1} p_i^{t-1}}_{Net-entry} . \tag{27}$$